Supplementary Material

Cross-scale feedbacks and tipping points in aggregated models of socio-ecological systems

Supplementary Material A: Case 1: Cascading through system elements

A.1 Model descriptions: connecting and perturbing variables

The equations of the '*threshold*' model read:

$$\frac{dS(t)}{dt} = r_S S(t)\left(1 - \frac{S(t)}{K_S}\right)\left(\frac{S(t)}{A_S} - 1\right) \quad \text{Eqn. (1a)}$$
$$\frac{dX(t)}{dt} = r_X S(t)X(t)\ \left(1 - \frac{X(t)}{K_X}\right)\left(\frac{X(t)}{A_X} - 1\right) - m_X X(t) \quad \text{Eqn. (1b)}$$
$$\frac{dZ(t)}{dt} = r_Z X(t)Z(t)\ \left(1 - \frac{Z(t)}{K_Z}\right)\left(\frac{Z(t)}{A_Z} - 1\right) - m_Z Z(t) \quad \text{Eqn. (1c)}$$

The below interpretations given for the symbols in Eqns. (1a-c) should be understood as such only for the threshold model.

Variables $S(t), X(t), Z(t)$ represent unspecified state variables of arbitrary units of measurement, where we can interpret $S(t)$ as some self-amplifying signalling variable, the production of $X(t)$ is promoted by $S(t)$, and $Z(t)$ in turn is promoted by $X(t)$. Variables have no specific values but note that the dynamics of the model may differ depending on the initial conditions $S(0)$, $X(0)$, and $Z(0)$, as will become apparent in the analysis of this model. Parameters r_S is an intrinsic growth rate of $S(t)$, while r_i with $i = \{X, Z\}$ are reaction or interaction rates. Parameters K_j with $j = \{S, X, Z\}$ represent maximal capacities for the respective state variables. In the case of K_S, it automatically represents the point where growth or influx of $S(t)$ is balanced with degradation or outflux. Parameters A_j with $j = \{S, X, Z\}$ represent implicit thresholds similar to what is known as the Allee effect in ecology (Stephens & Sutherland, 1999; Van Voorn et al., 2007). This effect, for instance, mimics growth inhibition in populations at low densities resulting from various negative effects like increased exposure to predation or reduced genetic variability. Here we assume some effects exist that prevent the amplification of interactions until sufficient mass or momentum has been accumulated; this could equally apply for some variable with a social interpretation, like the percentage of people with a certain opinion. Finally, parameters m_j with $j = \{S, X, Z\}$ represent some decay, break-down, or loss, i.e., if the signal or interaction is not maintained, the mass or momentum eventually is lost.

The '*switch*' model is an extension of the model by Wilhelm (2009) for bistability switches in cells. This model underlies, for example, signalling pathways for cell division and cell differentiation. The original model consists of two differential equations that can sustain two alternative stable steady states: an 'OFF' state, where nothing happens (e.g., no cell division), and an 'ON' state, where some process initiates (e.g., cell division), and which we consider here to be the preferable state. The model reads:

$$\frac{dA(t)}{dt} = A_m exp(-A_T t) - A_r A(t) - rA(t)B(t) \quad \text{Eqn. (2a)}$$
$$\frac{dB(t)}{dt} = rA(t)B(t) + 2k_1 C(t) - k_2 B(t)^2 - k_3 B(t)C(t) - k_4 B(t) \quad \text{Eqn. (2b)}$$
$$\frac{dC(t)}{dt} = k_2 B(t)^2 - k_1 C(t) \quad \text{Eqn. (2c)}$$

Again, the below interpretations given for the symbols in Eqns. (2a-c) should be understood as such only for the switch model.

Again, the state variables - here $A(t)$, $B(t)$, and $C(t)$ - have an abstract interpretation. Variables $B(t)$ and $C(t)$ appeared in the original model by Wilhelm, which consists of Eqn. (2b-c). In that model, the eventual state to which the system develops depends on the initial conditions $B(0)$ and $C(0)$: these need to be sufficiently large for the system to further develop to the ON state, otherwise it will decay back and remain in the OFF state. To jumpstart the bistability switch, we add eqn. (2a) with an additional variable $A(t)$ that interacts with $B(t)$. Note that time variable t appears explicitly in Eqn. (2a), which describes a pulse. The pulse first increases and then dies away; for example, it could mimic the waxing and waning of the public reaction to some event. While the pulse peaks, it may push $B(t)$ to a sufficiently high level for the switch to develop to the ON state. Parameters A_m, A_T, and A_r are all required to describe a pulse of variable $A(t)$: A_m affects the peak of the pulse, A_T is a time-dependent decay rate of the pulse, and A_r is a density-dependent decay rate. Parameter r indicates the interaction rate between $A(t)$ and $B(t)$. Note, that for $r = 0$ the switch model reduces to the original model by Wilhelm (2009). Finally, parameters k_i with $i = \{1,2,3,4\}$ are reaction rates.

A.2 Supplementary model analysis and results: threshold model Eqns. (1a-c)

The threshold model displays bistability (i.e., two stable equilibrium states). Depending on the initial conditions $S(0)$, $X(0)$, and $Z(0)$ the system evolves to a zero state in which all variables have collapsed or to a positive steady state, which we assume to be the desired state for transitioning.

The development to a positive steady state follows a cascading path in which first variable $S(t)$ rises, followed by $X(t)$, and finally $Z(t)$. The two displayed scenarios in the main text differ only in the initial condition $S(0)$; one could interpret this as the system being primed to tip, and it just requires a minor push across the threshold, which is represented by a small increase in $S(0)$. A further analysis of the model reveals multiple steady states, several of which are stable. The substitution of the parameter values that were used to create the simulations in Eqns, (1a-c), and then solving the equations for zero results in fifteen possible steady states. To determine the stability of each steady state, we first determine the 3-by-3 Jacobian matrix J containing the partial derivatives $\delta f_i/\delta x_j$, where f_i with $i = \{S, X, Z\}$ are Eqns. (1a-c), respectively, and $x_j = \{S, X, Z\}$ are the state variables; note that several partial derivatives evaluate to zero. Next, for each steady state we substitute the numerical steady state value in the Jacobian matrix to determine the eigenvalues of the respective steady state, which will tell us whether the steady state is stable or not. Four solutions have three negative eigenvalues and thus are stable steady states. These are $(S, X, Z) = (0,0,0)$, $(1,0,0)$, $(1,0.974,0)$, and $(1,0.974,0.974)$; the first and the last of these steady states also appear in Fig. 3. Besides the option for cascading, in which all state variables tip to their positive state one after the other, there is the possibility for tipping each of the state variables to a positive state separately, following the above ordering. This becomes clearer from a two-dimensional phase portrait of $S(t)$ and $X(t)$, assuming $Z(t) = 0$.

Supplementary Material B: Diet model details

This supplementary material outlines more details of the diet model. Table B1 outlines key assumptions in the model, and Tables B2-B6 outline the various components of the model - parameters, stocks, variables, and flows (following AnyLogic norms - Grigorvey, 2021).

B.1 Model Assumptions

Table B.1: Diet model key assumptions.

Model structure assumption	Explanation or data
Simulation steps represent one year and simulations run up to 50 iterations (i.e., representing 50 years)	Appropriate timeframe over which to model these SES dynamics.
The initial population size is set to a unitless reference level of 100. Based on the Our World In Data figures (Ritchie et al. (2023), the population growth increases but at a slowing rate during the next 50 years (from 0.82% in 2021 to 0.16% in 2071).	https://ourworldindata.org/future-population-growth
It is assumed that 10% of the protein demand by consumers can only be met by meat-based proteins.	We assume a subgroup of people will always be unwilling to switch to fully plant-based proteins.
For model initialization, we assume a distribution of 80% meat-based and 20% plant-based protein demand.	Based on: "What is notable in the above-mentioned data is that while the segment of vegans or vegetarians are at maximum at 10% of the population, consumers who regard

	themselves as flexitarian or are interested in reducing meat consumption are approximately 30–40% of the population." (Aschemann-Witzel et al., 2020) We assume flexitarian diet is equivalent to one-third vegetarian and the remainder meat-based.
Agricultural land is assumed to be divided into either land for meat-based or plant-based protein production. This ignores land used for products like biofuels, timber, or providing space for nature conservation, renewable energy production, recreation or tourism, etc.	Outside the scope of this model and its purpose.
While the change of land use for meat-based to plant-based proteins and vice versa is allowed in the model, we assume that 30% of the available land is unsuitable for plant-based production and can only be used for meat-based protein production methods such as extensive livestock or insect farming.	This is a relatively optimistic estimate based on the discussion of land use here https://ourworldindata.org/land-use Ritchie, H. & Roser, M. (2013)
For model initialization, we assume a distribution of 75% land for meat-based and 25% for plant-based protein production.	Based on Aschemann-Witzel et al., (2020)
The initial value for arable land is set to 100, but the value may increase with demand, assuming that the availability of natural land that can be cultivated is not the limiting factor.	We acknowledge that in reality land is a limited resource and the availability of arable land is finite. For the purpose of this conceptual model, we further ignore details around the investment requirements to change land use resulting from new equipment, supplies, management, personnel, and more.

Tables B2-B6 outline the various components of the model - parameters, stocks, variables, and flows (following AnyLogic norms - Grigorvey, 2021).

B.2 Diet Model general information

Table B.2: Diet model general design information.

Name	Value
General	
Model time units	years
System Dynamics solver	
Differention Equations Method	Euler
Algebraic Equations Method	Modified Newton
Mixed Equations Method	RK45+Newton
Absolute accuracy	1.0E-5
Time accuracy	1.0E-5
Relative accuracy	1.0E-5
Fixed time step	0.001
Advanced	
Java package name	diet_Model

Table B.3: Parameters information.

Parameters	Description	Default Values	Unit
Personal_Preference	A composite and aggregated parameter, this is intended to represent preferences such as environmental concerns, sustainability-related motives, and animal welfare concerns (Aschemann-Witzel et al., 2020), next to economic situations, religious reasons and ethical reasons for people to choose for a particular diet.	0.2	Unitless
Tech_Policy_Change_Plant	These are both composite parameters that represent available technologies, such as new technologies to produce plant-based protein products, subsidies, taxes, and policies that support or discourage production or consumption of either of the two protein alternatives.	0.5	Unitless
Tech_Policy_Change_Meat		0.1	Unitless
Consideration_Meat	Represents landowner and farmer knowledge and predisposition about producing specific crops or keeping livestock, social norms, and environmental concerns.	2	Unitless
Consideration_Plant		1	Unitless
Normative_factor_meat	Represent the demand resulting from social norms and governmental policy	0.066	Unitless
Normative_factor_plant		0.056	Unitless

Table B.4: Stocks information.

Stock	Initial Value	Unit	Explanation based on model assumption
Meat_Market_Capacity	80	Share of market capacity	All the market capacity is 100 in the first timestep, divided to 80 units Meat_market_capacity and 20 units Plant_based_Market_Capacity . During the simulation the capacity of the market between these two stocks change based on the demand that market receives and the available technology and production. The overall capacity of the market might go over 100 which is a translation of market expansion during time in real life.
Plant_based_Market_Capacity	20	Share of market capacity	
Meat_based_demand	80	Share of demand	All the demand for protein diets are 100 units in the first timestep, divided to 80 unit meat demand and 20 unit plant demand. During the simulation the demand between these two stocks changed based on people's preference and population . The overall demand might go over 100 if the population growth rules it.
Plant_based_demand	20	Share of demand	
Land_for_MeatProduction	75	Unit of land	The initial value for arable land is set to 100, but the value may increase with demand, assuming that the availability of natural land that can be cultivated is not the limiting factor. In the first time step the land is divided into 75 units for meat production and 25 for plant-based protein, this assumption is based on Aschemann-Witzel et al., (2020).
Land_for_Plant_based_Food	25	Unit of land	
Population	100	Block of people	The initial value of the people is set to 100 unit and it change with population growth rate

Table B.5: Variables information.

Dynamic Variable	Formula	Unit
Decision_Meat_Land	(Meat_Market_Capacity-Land_for_MeatProduction)>0? (Consideration_Meat)*(Meat_Market_Capacity-Land_for_MeatProduction) : 0	Unit of land
Decision_plant_land	(Plant_based_Market_Capacity-Land_for_Plant_based_Food)>0? (Consideration_Plant)*(Plant_based_Market_Capacity-Land_for_Plant_based_Food) : 0	Unit of land
Decision_Meat_use	(((1-Personal_Preference)+Normative_factor_meat)*Meat_Market_Capacity)/100<0.9? (((1-Personal_Preference)+Normative_factor_meat)*Meat_Market_Capacity)/100:0.9	Share of demand
Decision_Plant_Use	((Normative_factor_plant*Plant_based_Market_Capacity)/10)+Personal_Preference<0.9?((Normative_factor_plant*Plant_based_Market_Capacity)/10)+Personal_Preference : 0.9	Share of demand
Growth_Rate	Get_Growth_Rate(time())	block of people /year

Table B.6: Flows information.

Flow	Formula	Unit
flow	(Population*(Decision_Plant_Use))>0?(Population*(Decision_Plant_Use)): 0	Share of demand/year
flow1	Population * (Decision_Meat_use)>0? Population * (Decision_Meat_use):0	Share of demand/year
flow2	Meat_Market_Capacity-Meat_based_demand>0 ? ((Plant_based_demand-Plant_based_Market_Capacity>0) ? Meat_Market_Capacity-Meat_based_demand : 0) :0	Share of market capacity/year
flow3	Plant_based_demand>0 ? Plant_based_demand :0	Share of demand/year
flow4	Meat_based_demand>0 ? Meat_based_demand :0	Share of demand/year
flow5	Plant_based_Market_Capacity- Plant_based_demand>0 ? ((Meat_based_demand- Meat_Market_Capacity>0)? Plant_based_Market_Capacity- Plant_based_demand: 0):0	Share of market capacity/year
flow8	Decision_Meat_Land>0 ? Decision_Meat_Land :0	Unit of land/year
flow9	Land_for_MeatProduction>30 ? (decision_plant_land>0 ? decision_plant_land :0):0	Unit of land/year
flow11	Land_for_MeatProduction<90 ? ((Meat_based_demand - Meat_Market_Capacity)* (1+Tech_Policy_Change_Meat)>0 ? (Meat_based_demand - Meat_Market_Capacity)* (1+Tech_Policy_Change_Meat) : 0):0	Share of market capacity/year
flow12	Decision_Meat_Land>10 ? Decision_Meat_Land :0	Unit of land/year
flow13	decision_plant_land>10 ? decision_plant_land :0	Unit of land/year
flow14	Land_for_Plant_based_Food<90 ?((Plant_based_demand-Plant_based_Market_Capacity)*(1+Tech_Policy_Change_Plant)>0 ? (Plant_based_demand-Plant_based_Market_Capacity)*(1+Tech_Policy_Change_Plant):0) : 0	Share of market capacity/year

annual_population_growth	(Growth_Rate*Population)/100	Block of people/year

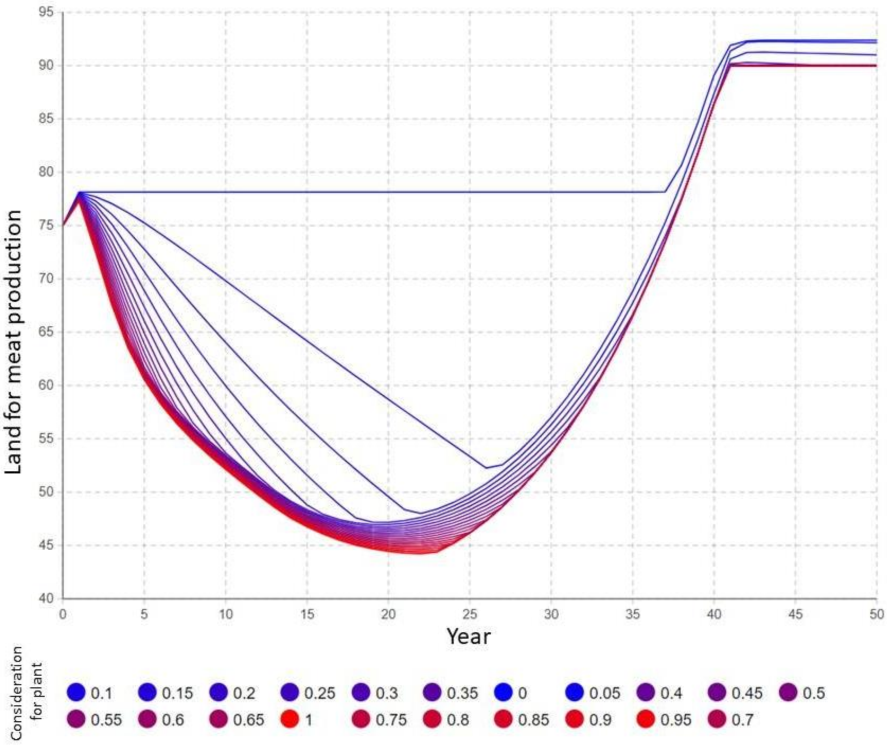

Figure B.1: Sensitivity Analysis of land for meat production stock versus changes in Consideration for plant parameter.

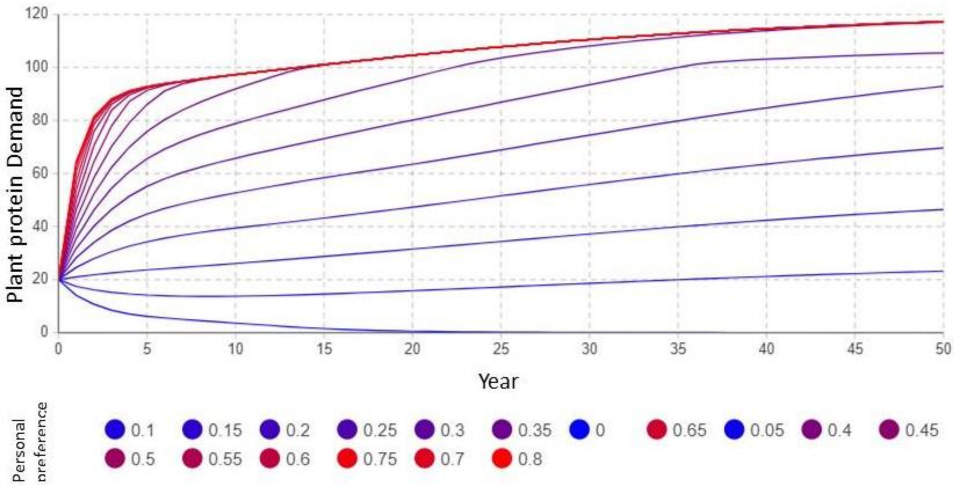

Figure B.2: Sensitivity Analysis of Plant Protein Demand stock versus changes in Personal preference parameters.

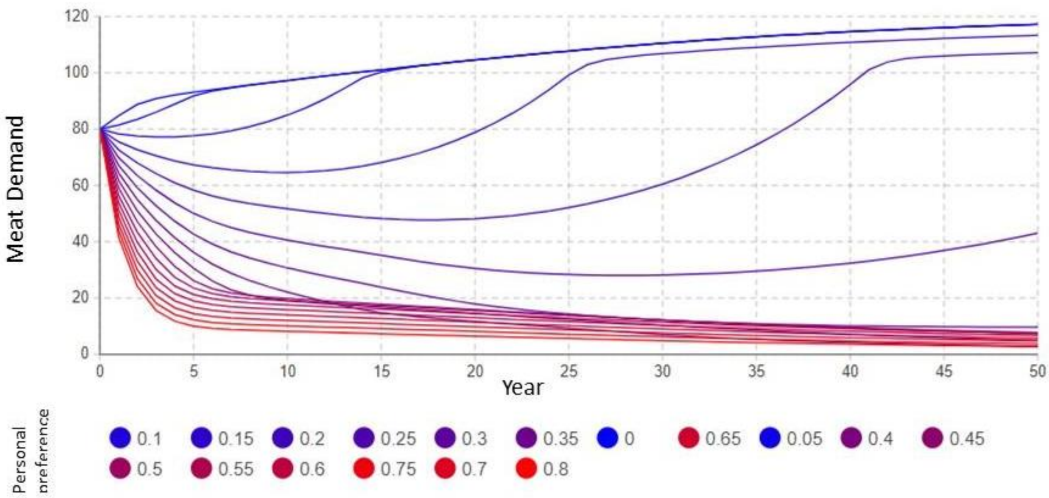

Figure B.3: Sensitivity Analysis of Meat Demand stock versus changes in personal preference.

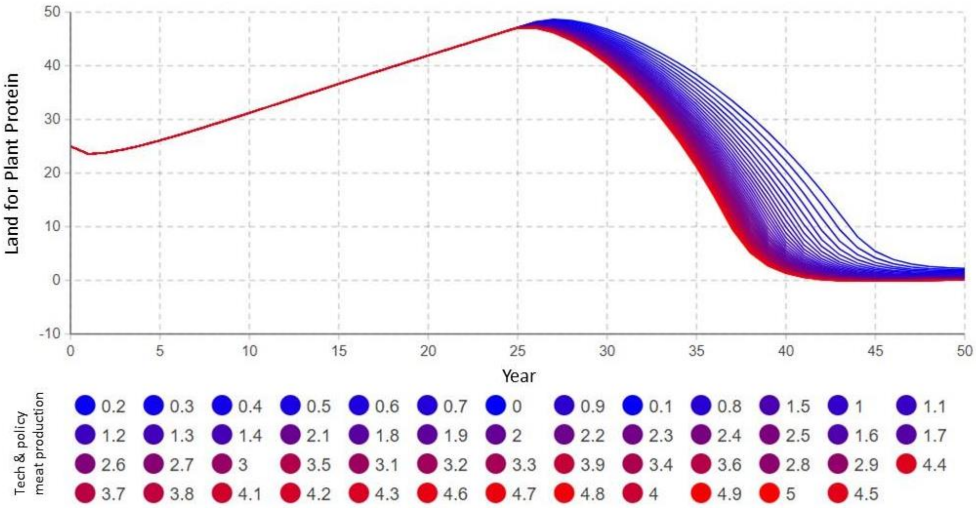

Figure B.4: Sensitivity Analysis of Land for Plant Protein stock versus changes in Tech and policy of meat production parameter.

Supplementary Material C: Case 3: A model of pyric herbivory in North American rangelands.

Rangelands cover approximately one third of the earth's land area, with at least one billion people dependent on these lands for their livelihoods (Follett and Reed, 2010). Most of the world's rangelands have been degraded by inappropriate land use practices (Millennium Ecosystem Assessment, 2005), primarily overgrazing by livestock (Teague et al., 2015). Overgrazing coupled with suppression of fire, exacerbated by global changes in atmospheric CO2, temperature, and rainfall, have facilitated continued encroachment of woody plants in what formerly were more open grasslands. Research suggests that proper management of the combination of fire and grazing (pyric herbivory) at the local level can mitigate woody plant encroachment. In the figure and table below, we present the causal relationships and associated equations, respectively, that describe our simple SES model representing pyric herbivory on a hypothetical cattle ranch in the rangelands of the southern Great Plains of North America.

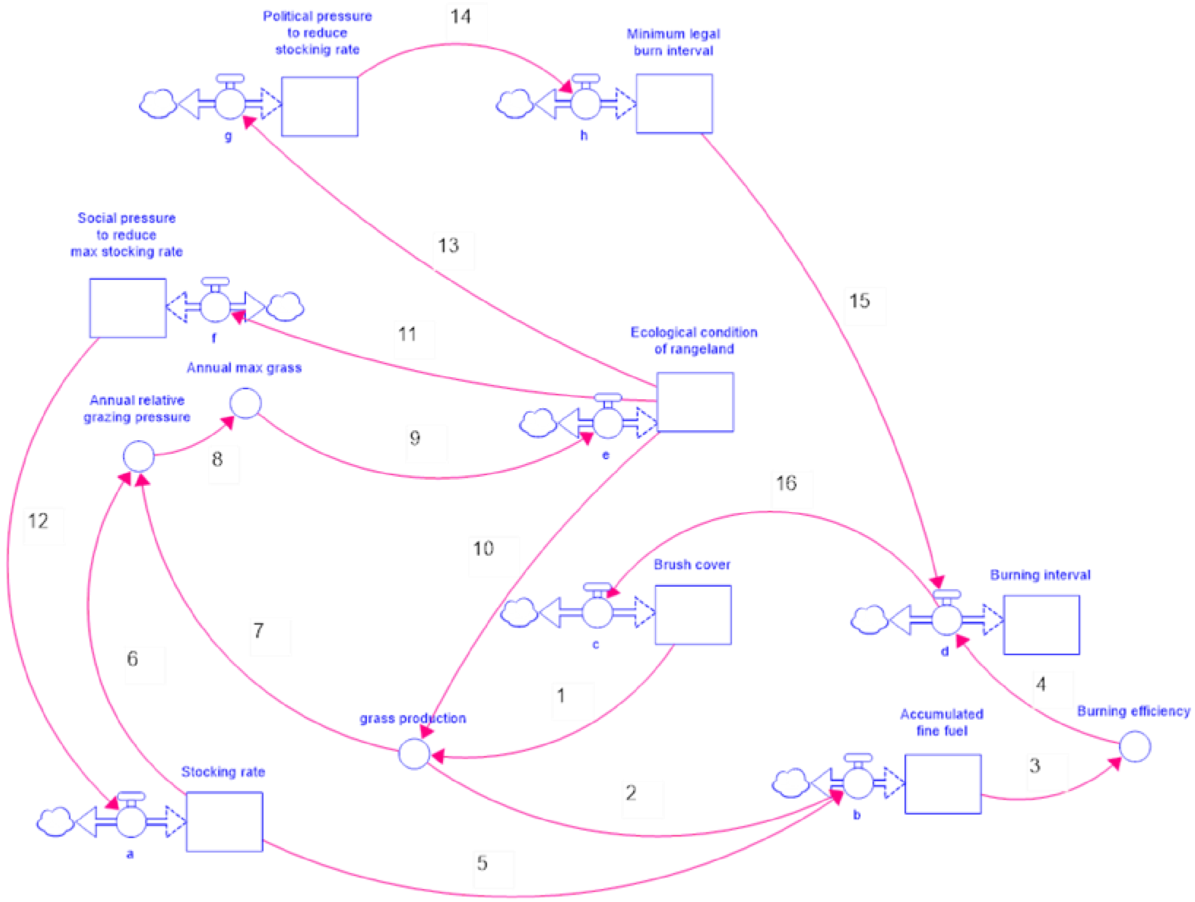

Figure C.1: Box and arrow diagram of the causal relationships represented in the pyric herbivory system. (1) Increased brush cover lowers grass production (due in large part to the shading effect on grasses caused by the increased canopy cover of woody vegetation). (2) Decreased grass production decreases accumulated fine fuel (decreases accumulation of dry, flammable dead grass). (3) Decreased fine fuel decreases burn efficacy (due to insufficient fine fuel to ignite a fire intense enough to burn brush). (4) Decreased burn efficacy increases brush cover (due to failure to periodically reduce brush sufficiently). (5) Increased stocking rate decreases accumulated fine fuel (via consumption of more live grass before it can senesce into dead grass). (6) Increased stocking rate increases annual relative grazing pressure (increases the ratio of grass consumption to grass production). (7) Increased grass production decreases annual relative grazing pressure (decreases the ratio of grass consumption to grass production). (8) Increased annual relative grazing pressure decreases annual max grass (decreases the maximum standing crop of grass, which commonly is used as an indicator of annual grass production). (9) Decreased annual max grass indicates decreased ecological condition of rangeland (via its indication of lowered annual grass production). (10) Decreased ecological condition of rangeland decreases grass production. (11) Decreased ecological condition of rangeland increases social pressure to reduce max stocking rate (to increase ecological condition via reduction of annual relative grazing pressure). (12) Increased social pressure to reduce max stocking rate decreases stocking rate. (13) Decreased ecological condition of rangeland increases political pressure to reduce minimum burn interval (to increase ecological condition via more frequent burns to reduce brush cover). (14) Increased political pressure to reduce minimum burn interval decreases minimum legal burn interval. (15) Decreased minimum legal burn interval decreases burn interval. (16) Decreased burn interval decreases brush cover.

Table C.1: Summary of the parameter values and functional relationships represented in the pyric herbivory model.

LiveGrass(t+1) = LiveGrass(t) + [max-grass-growth × LiveGrass(t) × den-dep-grass-growth-factor(t) × brush-shading-grass-growth-factor(t)× EcologicalCondition(t)]- [LiveGrass(t) × prop-grass-loss-to-herbivory(t)]

max-grass-growth = 0.7

den-dep-grass-growth-factor(t) = 1 - 0.01 × LiveGrass(t)

brush-shading-grass-growth-factor(t) = 1 - 0.01 × Brush(t)

prop-grass-loss-to-herbivory(t) = stocking-rate(t) × 0.02

DeadGrass(t+1) = DeadGrass(t) – [DeadGrass(t) × decomp-rateG]

decomp-rateG = 0.1

if Month = 1: DeadGrass(t+1) = DeadGrass(t) + LiveGrass(t); LiveGrass(t+1) = 1

Brush(t+1) = Brush(t) + [max-brush-growth × Brush(t) × den-dep-brush-growth-factor(t)] – [Brush(t) × prop-burn-loss-brush(t)]

max-brush-growth = 0.05

den-dep-brush-growth-factor(t) = 1 - 0.01 × Brush(t)

if burn = 1: prop-burn-loss-brush(t) = FineFuel(t) / 100, else prop-burn-loss-brush(t) = 0

FineFuel(t) = LiveGrass(t) + DeadGrass(t)

EcologicalCondition(t+1) = EcologicalCondition(t) – [grazing-pressure(t) × 0.001]

grazing-pressure(t) = stocking-rate(t) / LiveGrass(t)

if stocking-rate = 1: EcologicalCondition(t+1) = EcologicalCondition(t) + 0.01

if stocking-rate(t) > socially-desired-SR(t): stocking-rate(t+1) = socially-desired-SR(t), else stocking-rate(t+1) = stocking-rate(t) [for Scenario 1], or stocking-rate(t+1) = stocking-rate(t) + 1 [for Scenario 2]

socially-desired-SR(t) = 20 – [2 × SocialPressureMaxStockingRate(t)]

SocialPressureMaxStockingRate(t) = 10 - 10 × EcologicalCondition(t)

if burning-interval(t) ≠ MinLegalBurnInterval(t): burning-interval(t+1) = MinLegalBurnInterval(t), else burning-interval(t+1) = burning-interval(t)

MinLegalBurnInterval(t) = 5 – [2 × PoliticalPressureMinBurnInterval(t)]

PoliticalPressureMinBurnInterval(t) = 10 – [10 × EcologicalCondition(t)]