

Cross-scale feedbacks and tipping points in aggregated models of socio-ecological systems

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Abstract

Many researchers have called for more consideration of cross-scale dynamics in models of socio-ecological systems, but this is a fundamentally difficult thing to do. Focussing on cross-scale feedbacks and tipping points, this paper uses three example models to demonstrate and reflect on how cross-scale dynamics can be incorporated into *aggregate* models. Tipping points - where a small perturbation can lead to a qualitative change in a system - are generally the result of nonlinear feedback mechanisms. These feedback mechanisms often operate on different levels within or across scales. Tipping points occurring on one level or scale may cascade across to others. Interest in these types of cross-scale feedbacks and tipping points is high, but consideration of how to model them is underdeveloped. The representation of cross-scale feedbacks and tipping points in aggregated models of socio-ecological systems remains a critical challenge for modellers, with implications for the types of models and policy advice that can be developed. We present three case studies to demonstrate and reflect on how cross-scale feedbacks and tipping points can be represented and analysed in these models. Two key themes emerge from our reflections: (i) the variety and trade-offs in ways to explore and present model behaviour, using tools such as scenario analysis and phase portraits; and (ii) the subjectivity inherent in considering and implementing scale, in aggregated models.

Keywords

Cross-scale dynamics, Socio-ecological systems, feedbacks, tipping points, aggregated models

Code availability

The equations and parameter values for the Case 1 and 3 models are provided in figures and the Supplementary Material. The model files for Case 2 can be found <https://github.com/bapeterj/DietModelSESMPaper>. For Case 1, readers should be able to recreate the simulations and phase portraits in any numerical package of their choice, e.g., R (with deSolve and Phaser), Python, Fortran, Jupyter notebook, Mathematica, Matlab, Maple, C++, etc. For Case 3, readers should be able to recreate the simulations using NetLogo.

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1. Introduction

In recent years, there has been a growing interest in the concept of tipping points in the literature on climate, earth systems, and Socio-Ecological Systems (SES) (Manjana et al., 2018; Winkelmann et al., 2021; Lenton et al., 2022). Initially, the focus was on describing the harmful aspects of tipping points in these systems which might be reached because of climate change or other human impacts (e.g., permafrost melting). In particular, as SES are becoming more connected (Troell et al., 2014), the concern arose that if these systems contain tipping points, that (unforeseen) perturbations at small scales may result in systems undergoing a ‘perfect storm’ of cascading collapses, i.e., a large-scale ‘domino effect’ of collapses of SES (Biggs et al., 2011; Helbing, 2013; Kloze et al., 2020; Van Voorn et al., 2020). Should such a transition occur at an Earth-System level, it could affect central social and economic systems and quickly degrade humanity’s condition (Homer-Dixon et al., 2015).

There is now a parallel and active stream of research exploring desirable, or ‘positive’, tipping points in SES, i.e., tipping points that may be triggered to promote a transition to a more desirable state. Such tipping points might help, or indeed be required, to achieve positive climate and environmental outcomes. This literature has sought to explore what defines a ‘social tipping point’ (e.g., Winkelmann et al., 2021; Füllsack et al., 2021), how they work or what interventions might be developed to reach, create, or trigger tipping points (e.g., Farmer et al., 2019; Lenton, 2020; Hepburn et al., 2020; Lenton et al., 2022; Mealy et al., 2023), and has begun to consider the cross-scale dynamics, specifically upward-scaling cascades of tipping points, we might hope to see (e.g., Sharpe & Lenton, 2021; Systemiq, 2023).

This research has tended to focus on socio-technical systems (e.g., the transition to a low carbon economy), but earlier work did attempt to consider SES (Lenton et al., 2022). However, focus was on tipping points within individual local SES, or on how change at higher-scales affects local systems, but did not explore how change and tipping points in local systems may contribute to higher-level changes (Hughes et al. 2013; Lenton, 2020) or how tipping processes at different scales interact.

Marten (2005) developed the first database of ‘EcoTipping points’ which documented examples of positive change affected by tipping points, the most famous example being the recovery of Apo Island fisheries in Japan. Rocha et al. (2018) built causal networks of regime shifts in ecosystems (or more specifically, causal loop diagrams), using the extensive literature review in Biggs et al. (2018), describing the connections between different regime shifts in ecosystems. However, it excludes social and economic drivers or explicit spatial or social scales. Conversely, Lenton (2020), in his review paper, describes some of these cross-scale issues at a conceptual level without formalising these ideas with examples or consideration of how to model these cross-scale cascading dynamics.

In contrast, the sustainability literature has specifically focused on cross-scale dynamics of institutions and technologies. Lam et al. (2020) discuss frameworks of amplification processes for sustainable transitions and focus on scale dynamics as one of those frameworks. Abson et al. (2017) use restructuring as a conceptual tool to think about how cross-scale dynamics of institutions and the law needed to be leveraged to create more positive tipping points. Ahlborg et al. (2019) merge these concepts using technology as a framework and posit that understanding institutions as a technology is one way to make tipping points more actionable across scales.

This paper seeks to complement and build on these parallel literatures, and the recent push for more focus on cross-scale dynamics in SES, by considering how we can better model cross-scale feedbacks and tipping points in SES. Since fine-grained and spatially explicit models, such as agent-based models, inherently represent scale (either spatially, or socially through the representation of different types of agents and actors), we focus on representing cross-scale dynamics in *aggregated* models. Aggregate models of SES are a well-established and valuable component in the modelling and analysis landscape. However, they do not inherently represent or consider scale and so present a more serious challenge for modellers of SES looking to consider cross-scale dynamics.

By ‘aggregate model’ we are referring to models which routinely or primarily aggregate actors, measures, or concepts in systems to model them. For example, System Dynamics models, or equation-based models, as opposed to individual-based or agent-based models, which do not typically aggregate the parts of systems they are focussed

on (though they may represent meso or macro level actors or processes in aggregated forms). In practice, there may be aggregated models with different components of sub-models that do attempt to represent different scales or levels within a system (they may be referred to as ‘integrated’ though this term is used in many different ways). Nonetheless we believe cross-scale dynamics remains a fundamentally open question for these models in a way which is not the case for agent-based or spatially explicit models.

Before we proceed, we need to pin down some definitional issues. On scales, we use ‘levels’ and ‘scales’ interchangeably to refer to the different points within a scale (e.g., ‘local’, ‘regional’, and ‘national’ might be the scales or levels within a geographic scale). On tipping points, we take the definition to be a critical threshold in a parameter of a system, at which a small change produces a qualitatively different structure or new stable state of that system perpetuated by feedback loops (Lenton et al., 2022). Feedback loops are causal mechanisms in which an output of a process has a causal impact on its inputs, which can either dampen the process (a balancing feedback) or strengthen it (a reinforcing feedback). The increasing importance of reinforcing feedback in a system is what often causes tipping points to be crossed. Tipping elements are the components of systems, which might contain tipping points. Cascading occurs when crossing a tipping point results in the approach or crossing of another tipping point in a different system or element. When these cascades happen from smaller to larger temporal and/or spatial scales and increase the likelihood of crossing more tipping points, they are termed ‘upward-scaling tipping cascades’ (Sharpe & Lenton, 2021). Vice-versa, when they move from higher to lower levels, we can term them ‘downward-scaling’’. Figure 1 outlines these conceptual issues and provides more examples.

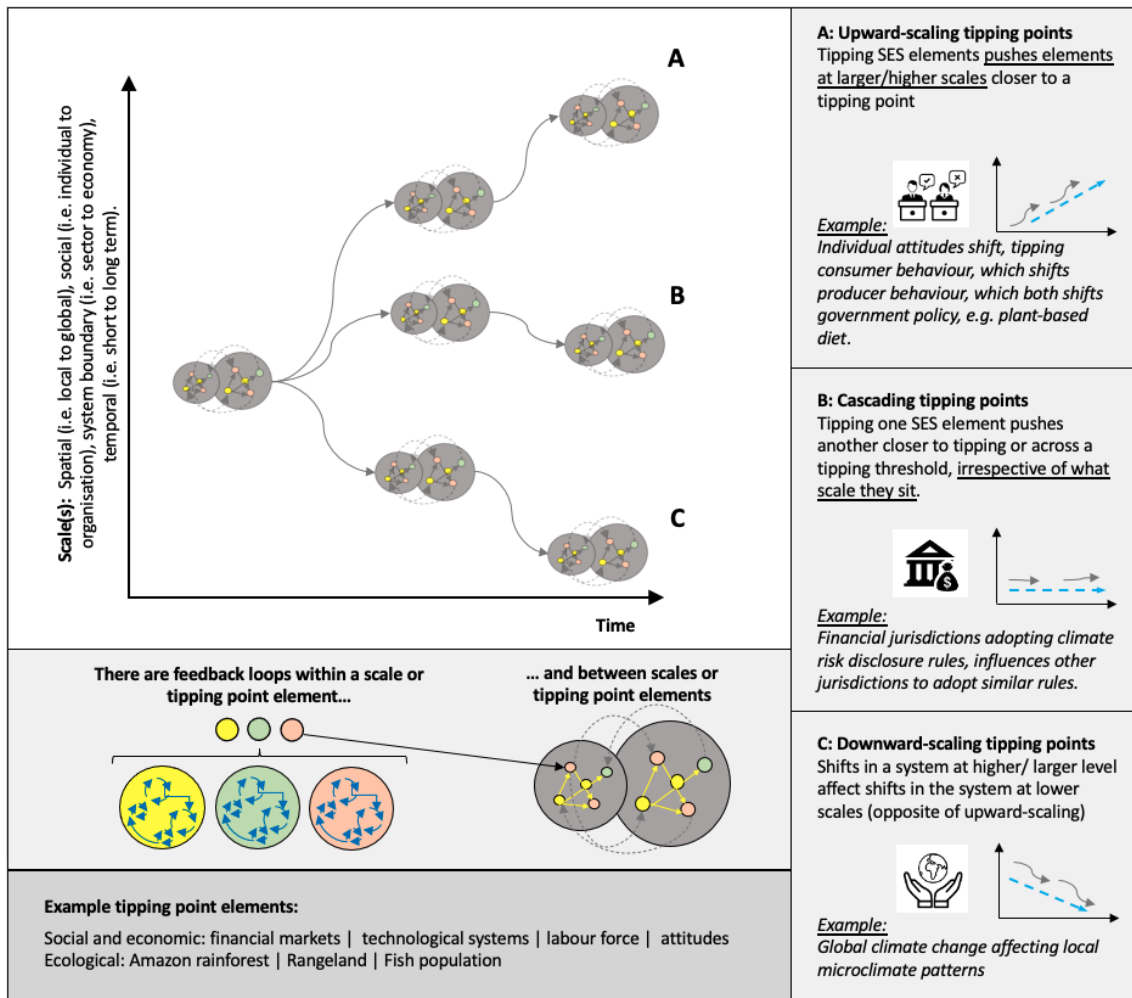


Figure 1: Modelling cross-scale feedbacks, tipping points, and cascades in socio-ecological systems.

To address the goals of this paper we develop three contrasting examples of how cross-scale feedbacks and tipping points can be represented in aggregated models of SES. Each example has in turn an increasing level of realism and application. The first example consists of two abstract conceptual models of linked differential equations to demonstrate the principle of cascading tipping points. The second and third examples consider the real-world SES of dietary choices and agriculture (a relatively abstract model with a strong social component) and burning strategies in land management (a more detailed model with a strong ecological component), respectively.

Each example is analysed with the following questions in mind, though the emphasis in each case varies between questions (note, this exploration is summarised in table form in the Discussion section):

1. How are cross-scale feedbacks and tipping points represented? (i.e., a model design question);
2. Under what conditions might we get cascading, upward-, or downward-, scaling tipping points in this system? (i.e., a model behaviour question);
3. What interventions could be made to promote desirable cascades or scaling, or prevent unwanted ones? (i.e., a policy question); and
4. How was the analysis of the model focused on understanding the cross-scale issues? (i.e., a model analysis question).

The three cases and the analysis they present are varied in nature. This is intentional, to help see how a variety of aggregated models can be considered from this cross-scale perspective. By conducting these three case studies, addressing these questions, and reflecting on the modelling process and analysis, we aim to directly address our goal of better understanding how to consider cross-scale feedbacks and tipping points in aggregate models of SES. Two key themes emerge from our discussion: (i) the variety and trade-offs in ways to explore and present model behaviour, using tools such as scenario analysis and phase portraits; and (ii) the subjectivity inherent in considering and implementing scale, in aggregated models. We believe researchers and modellers who work with aggregated models of SES should find the paper useful to directly consider how to incorporate scale into their work. We hope modellers working with disaggregated or spatial models, or the users of SES models more broadly, will find the paper useful in a more diffuse way, to help them reflect on how scale, feedbacks, and tipping points are represented and analysed in their work.

2. Case 1: Cascading through system elements

This example presents two conceptual models with abstract mechanisms for generating a cascading series of switches without a particular real-life interpretation. Both models are based on differential equations. Such models are, for instance, commonly used in Systems Biology to describe mechanisms for generating cascading signals (Klipp et al., 2016). The first model, the ‘threshold’ model, shares important features with the feed-forward loop model (Kim et al., 2008) for the amplification of low-level, noisy signals through cascading processes occurring in living cells. It demonstrates how crossing a tipping point on one level can promote crossing a tipping point on another level. The second model, the ‘switch’ model, is an extension of the model by Wilhelm (2009) for bistability switches in cells. It demonstrates how a temporary signal can lead to a permanent switch in a coupled feedback system. The state variables are assumed to be homogeneous and neither of the two models have explicit scales, but they could be interpreted to have scales given the context. Both models represent feedback by law-of-mass-action interaction terms. In the ‘threshold’ model, the tipping point is explicitly stated by the threshold formulation. In the ‘switch’ model, the tipping point emerges from a mutual promoting interaction.

2.1 Threshold and switch model design

The model equations, values of parameters and initial conditions, and description of the model analysis can be found in Supplementary Material A; a schematic overview is given in Figure 2. The models can be implemented in any software that is capable of the numerical simulation of differential equations; here we used Maple (Maplesoft, 2021) for the simulations and model analysis.

The ‘*threshold*’ model consists of three variables $S(t), X(t), Z(t)$, where we can interpret $S(t)$ as some self-amplifying signalling variable, the production of $X(t)$ is promoted by $S(t)$, and $Z(t)$ in turn is promoted by $X(t)$. The key mechanism in the threshold model is similar to what is known as the Allee effect in ecology (Stephens & Sutherland, 1999; Van Voorn et al., 2007). We assume that some effects exist that prevent the amplification of interactions until sufficient mass or momentum has been accumulated, i.e., a threshold is crossed.

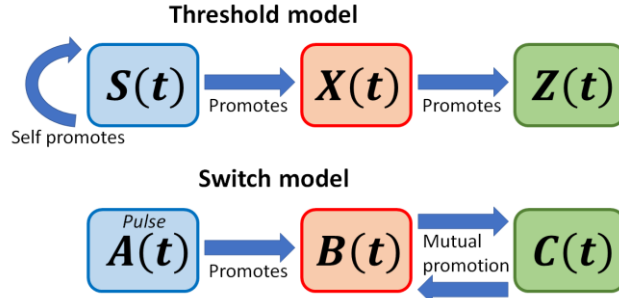


Figure 2: Schematic overview of the threshold and switch models. Note that variable $A(t)$ in the switch model indicates a pulse.

The ‘*switch*’ model is an extension of the model by Wilhelm (2009) for describing signalling pathways for cell division and cell differentiation. The original model consists of two differential equations and allows for two alternative stable steady states: an ‘OFF’ state, where nothing happens (e.g., no cell division), and an ‘ON’ state, where some process initiates (e.g., cell division), and which we consider here to be the preferable state. We add an additional variable $A(t)$ to jumpstart the bistability switch.

2.2 Threshold and switch model analysis

For the threshold model, we consider two scenarios that differ only in $S(0)$, the initial value of $S(t)$. In the first scenario the initial condition is insufficient to jumpstart a cascading sequence, and all variables eventually drop to zero (Figure 3, left panel). In the second scenario the initial condition is marginally larger and sufficient to result in a cascading development to a positive steady state, which we assume to be the desired state for transitioning (Figure 3, middle panel).

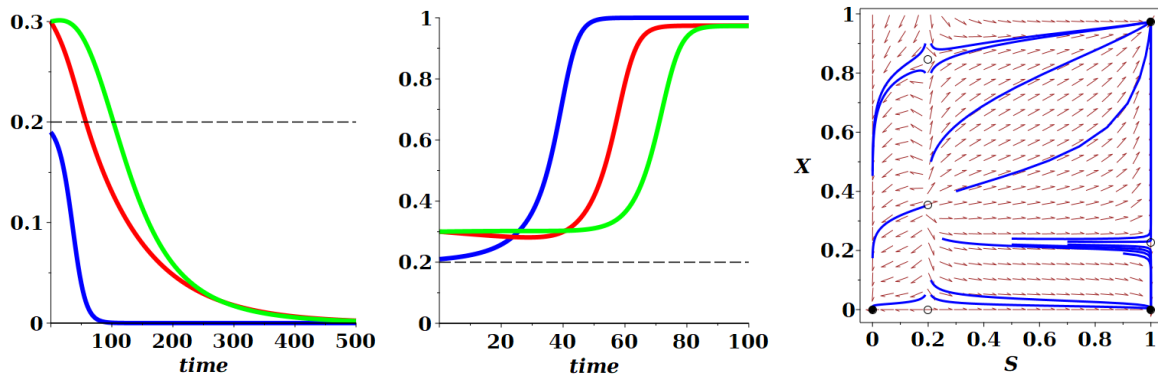


Figure 3: Simulations with the threshold model, Eqns. (1a-c), with $r_S = r_X = r_Z = 0.1$, $K_S = K_X = K_Z = 1$, $A_S = A_X = A_Z = 0.2$ (indicated by the dashed lines), and $m_X = m_Z = 0.01$. Depicted are $S(t)$ in blue (leading in time), $X(t)$ in red, and $Z(t)$ in green. Left panel: A cascading collapse with initial conditions $S(0) = 0.19, X(0) = 0.3, Z(0) = 0.3$. Middle panel: A cascade evolving to a positive steady state, under the same initial conditions except $S(0) = 0.21$. Right panel: a phase portrait for $Z(t) = 0$.

A further analysis of the threshold model reveals there are multiple steady states, four of which are stable, namely $(S, X, Z) = (0, 0, 0), (1, 0, 0), (1, 0.974, 0),$ and $(1, 0.974, 0.974)$. A phase portrait of Eqns. (1a-c), with $Z(t) = 0$ (Figure 3, right panel) clarifies the required conditions for cascading. Three of the four stable steady states are given in solid black (note, that the state $(1, 0.974, 0.974)$ cannot be displayed), and several unstable steady states are indicated

by open circles. Red arrows denote the direction field, while several orbits are given in blue. The depicted orbits show how the plane can be divided into three domains of attraction: one for each of the stable states $(0,0,0)$, $(1,0,0)$, and $(1,0.974,0)$. This implies that the cascading effect will only happen when the value of $S(0)$ is sufficiently large while also the values of $X(0)$ and $Z(0)$ are sufficiently large. We can interpret this as the system being primed to tip, and then any $S(0) > 0.2$ is sufficient to cascade the system to the desired state. Note, that $X(t)$ and $Z(t)$ decline if they are not being pushed by $S(t)$, so the time window for the push to result in a cascade is limited, with the length of the window dependent on the value of $X(0)$ and $Z(0)$: the larger these values, the larger the time window to push the system into a desirable cascade. This also means that as an intervention strategy $S(0)$ should be taken high enough.

The switch model also displays bistability like the threshold model, but in this case, it is an emergent result from the reaction equations. The initial conditions $B(0)$ and $C(0)$ need to be sufficiently large for the system to further develop to the ON state, otherwise it will decay back and remain in the OFF state. To jumpstart the bistability switch, we add an additional variable $A(t)$ that interacts with $B(t)$ and which describes a waxing and then waning pulse. While the pulse peaks, it may push $B(t)$ to a sufficiently high level for the switch to develop to the ON state. This is demonstrated by two simulations of the model. In one simulation, the coupling between the jumpstart variable and variable $B(t)$ is zero, and the initial conditions are insufficient to progress to the ON state (Fig. 4, upper left panel). In the other simulation, this coupling is large enough to trigger a cascade (Fig. 4, upper right panel).

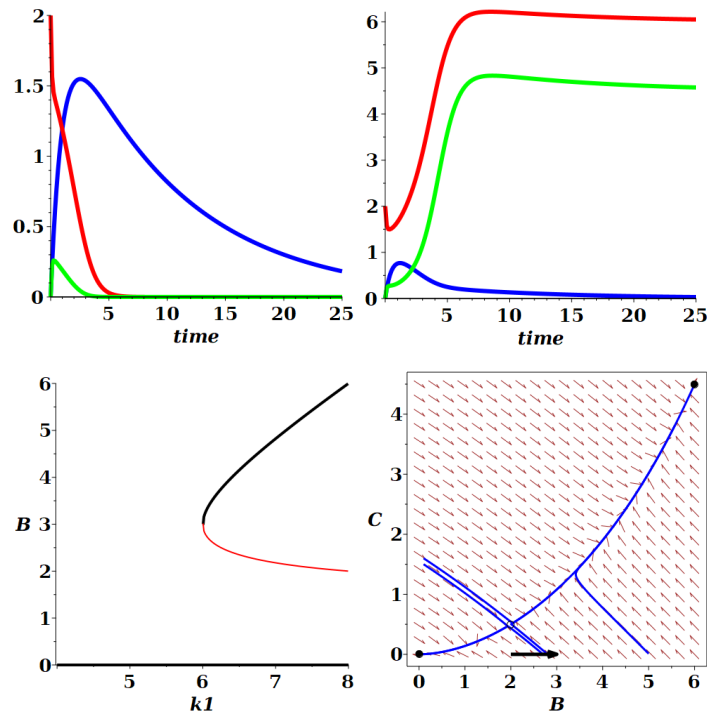


Figure 4: Simulations with the switch model, Eqns. (2a-c), with $A_m = 2$, $A_r = 1$, $A_T = 0.1$, $k_1 = 8$, $k_2 = 1$, $k_3 = 1$, $k_4 = 1.5$, and initial conditions $B(0) = 2$, $A(0) = C(0) = 0$. Depicted are $A(t)$ in blue (i.e., the pulse), $B(t)$ in red, and $C(t)$ in green. Upper left panel: With $r = 0$, the system is similar to the model by Wilhelm (2009). While there is some initial increase in $C(t)$, the value of $B(0) = 2$ is insufficient to initiate a switch to the ON state. Upper right panel: With $r = 0.75$, i.e., $A(t)$ and $B(t)$ are coupled, the pulse of $A(t)$ is sufficient to initiate a transition to the ON state. Lower left panel: A bifurcation diagram with the steady state value of B as function of parameter k_1 , keeping all other parameters fixed. At $k_1 = 6$ a tipping point occurs in which a positive stable state (at the left end of the upper black curve) merges with a positive unstable steady state (left end of the red curve). The latter is associated with a threshold for values of $k_1 > 6$. Lower right panel: A phase portrait of Eqns. (2b-c), with $A = 0$ and $k_1 = 8$. Two stable attractors $(0,0)$ and $(6,4.5)$ exist and are indicated by solid black, while an unstable steady state is indicated by an open circle. Red denotes the direction field, while orbits are given in blue. Based on the orbits we can approximate where the internal threshold lies for $C = 0$. The black arrow indicates how B should be increased (here $B > 2.8$ approximately) to achieve a switch from $(0,0)$ to $(6,4.5)$.

The bifurcation diagram (Fig. 4, lower left panel) and phase portrait (Fig. 4, lower right panel) clarify the conditions for cascading. For more on bifurcation analysis methods, see Kuznetsov, 2004; Van Voorn & Kooi, 2017. For a range of $k_1 > 6$, the model displays two alternative attractors: the OFF state, and the ON state. Each attractor has a separate domain of attraction. These domains of attraction are separated by a tipping point that is linked to the unstable steady state (indicated in red in Fig. 4, lower left panel). This tipping point needs to be crossed for the system to develop to the ON state. Without any push, the crossing of the tipping point only happens when the state variables are sufficiently large. With a push from a promoting variable, $A(t)$, the crossing of the tipping point is made easier. For values of $k_1 < 6$ the bistability disappears and only the OFF state remains, which explains that no cascade can be triggered. This implies a necessary condition for the cascading tipping point, namely that k_1 should be bigger than some minimal value. In addition, either the pulse $A(t)$ or the initial condition $B(0)$ should be large enough for the cascade to occur. Any intervention strategy to promote a cascade should be aimed at fulfilling these necessary conditions.

2.3 Threshold and switch model reflections

We have summarised the results of both models. Each variable represents an attribute at an arbitrary scale. Variables can be ecological or sociological in nature. The models demonstrate some essential mechanisms for generating cascading through tipping points that appear in the later examples. In both models, feedback is represented by law-of-mass-action dynamics, promoting interaction between the variables, which is essential to get a cascading response. In the threshold model, the tipping point is explicitly described by the formulation of an Allee-effect type of mechanism. In the switch model, the tipping point emerges from the mutual promoting interaction between two variables. In the threshold model, the minimal conditions for cascading are that all variables are higher than their respective threshold values; if this is the case, then one variable through law-of-mass-action interaction will push the next one until some positive steady state is achieved. Any intervention should be aimed at increasing each variable to such a value that the thresholds are all crossed. In the switch model, the parameter settings need to be adequate, and the jumpstart value needs to be high enough and well-timed for the mutual interaction to result in a positive steady state. Any intervention should be aimed at setting the parameters at adequate values and initiating a sufficient jumpstart pulse. The low-complexity nature of the models allows for a thorough analysis with the use of well-established methods (steady state analysis, phase portraits, bifurcation analysis) to understand the conditions for cascading. For models involving more realistic applications the additional use of sensitivity analysis methods is recommended, e.g., to quantify interaction effects (see Razavi et al., 2021 for a discussion on sensitivity analysis for environmental models for decision making).

3. Case 2: Shifting from meat to plant-based diets

Our second example, the diet model, is still relatively abstract but tailored to a specific real-life application, namely the potential transition from meat-based to plant-based protein production and consumption. Shifting to more plant-based diets has been suggested as an important part of the shift to a low carbon and more sustainable society, while also improving food security (Eker et al., 2019). Here, we present a model, referred to as the 'Diet model', developed explicitly to consider the various levels and scales involved in this topic and the feedbacks between them.

3.1 Diet model design

The diet model focuses on three thematic levels within a combined social and geographical scale: the demand for plant-based and meat-based proteins from individuals (a micro scale), the capacity of the market to produce plant-based and meat-based proteins (a meso scale), and the global land use for their respective production (a macro scale). The model was developed as a System Dynamics model using AnyLogic software (Grigorvey, 2021).

An overview of the diet model is given in Figure 5. The diet model shows an interconnected socio-environmental system in which there are many cross-scale feedbacks. For example, individual-level demand for meat and plant-based proteins is affected by population and personal preferences but also capacity in the market for each type of

protein. Market capacity is affected by technology and policy at market level, but also individual-level demand and global land use. There are feedbacks between each level in the model. The interactions and interconnections between variables, parameters, and stocks in the model are derived from data, literature, and logical assumptions. These assumptions have been documented in Table B1 in the Supplementary Material. In several instances, logical assumptions have been employed to establish connections within the model, such as the relationship between population and protein demand. It is a well-founded assumption that a higher population corresponds to an increased demand for protein, whether meat or plant-based. Moreover, the intricate interplay among variables including consumer demand, personal preferences, normative factors, and decision-making processes as well as market capacity and land use are primarily informed by research, particularly drawing from the literature (Aschemann-Witzel et al., 2020; Chrysafiet al., 2022). This ensures that the model's structure and relationships are grounded in empirical findings and expert knowledge. Many assumptions and simplifications have been made to keep the complexity of the diet model in check. We recognize that many alternative model formulations and assumptions are possible and each different set of assumptions and formulations would impact the results. In Supplementary Material B, we list the important structural assumptions, variables, and model specification. To evaluate if these coupled feedback loops generate nonlinear emergent behaviour and cascading effect, we use simulations and sensitivity analysis.

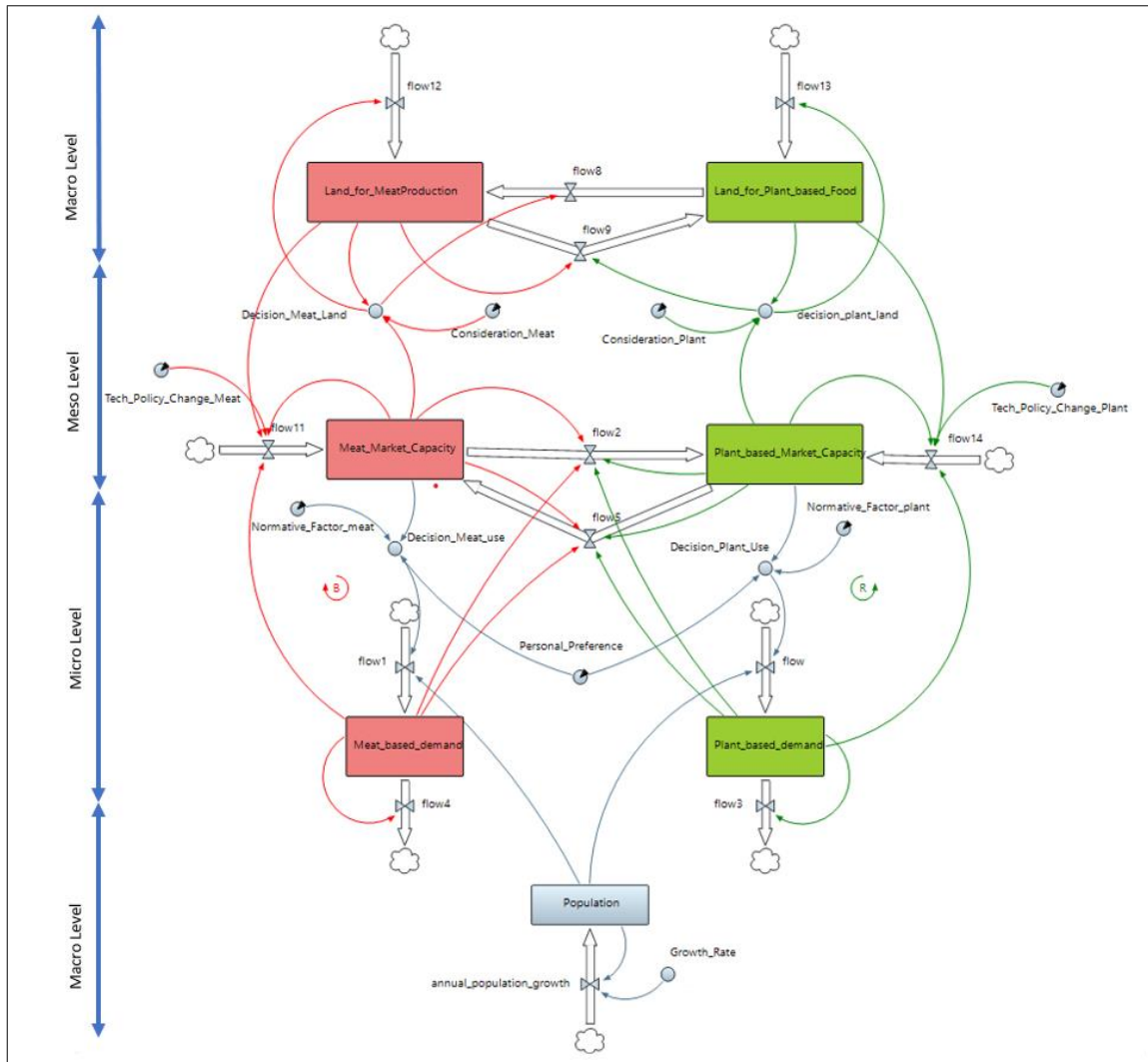


Figure 5: A stock-and-flow diagram of the diet model, created using AnyLogic software (Grigoriev, 2021) following standard stock and flow diagram iconography (structural assumptions, variables, and model specification are discussed in Supplementary Material B).

3.2 Diet model analysis

We present a two-stage analysis. In the first stage, we applied a sensitivity analysis in which we varied all parameters step-wise one at a time, i.e., we set all the factors constant except one - of which we step-wise vary the value - and then for each new value we run the model for 50 time steps (representing years) to quantify the changes in the output variables with respect to the changed factor value. This procedure is then repeated for each factor. The sensitivity analysis exposed several nonlinearities in the model behaviour of which the most notable ones are shown in Figure 6 and the Supplementary Material B. Here, we focus on one of these nonlinear behaviours in the model concerning a variable on individual decision making and demand for plant-based protein.

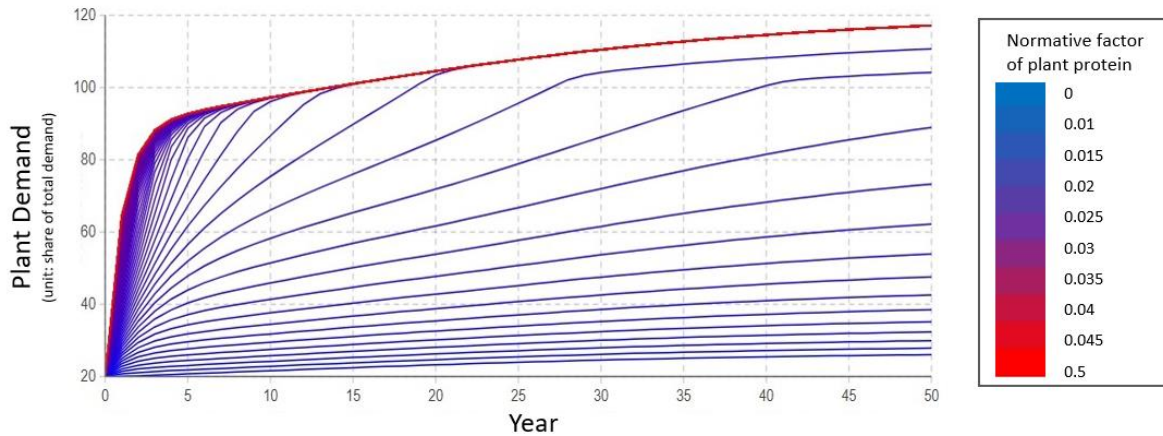


Figure 6: The sensitivity of the plant-based protein demand over time versus changes in normative factor of plant protein in the model. Y-axis shows demand for plant-based proteins, and x-axis shows time (year). The different lines represent different values of the normative factor of plant protein.

Figure 6 illustrates the sensitivity of plant-based protein demand over time, with the y-axis representing the demand for plant-based proteins and the x-axis depicting time in years. Each distinct line on the graph represents variations in the ‘normative factor’ of the plant protein weight parameter, highlighting its impact on the changing demand trends. ‘Normative factor’ represents the demand resulting from social norms and governmental policy. This factor is used in the calculation of plant-based protein demand, alongside market capacity and personal preferences (please refer to Supplementary Material for the equations). For low values (0-0.05) of the normative factor the demand marginally increases and remains low. For 0.05-0.1 values the simulations rise faster but all tail off at around 3 years. For values greater than 0.1, it becomes evident that the parameter no longer exerts a significant influence on the rate of demand increase. The rate of increase remains substantial until reaching a demand level of approximately 90, after which it increases with a slower rate.

Figure 7 illustrates the resulting response curve for the demand of plant-based protein concerning the normative factor. This response exhibits an exponential growth pattern as the normative factor increases from 0 to approximately 0.06, at which point it swiftly levels off to a demand just below 120. Beyond this point, the demand becomes constrained primarily due to limitations imposed by market capacity and available land. In subsequent simulations, we set the normative factor to 0.05. This clearly demonstrates the nonlinear response of the model to this parameter, a consequence of the interrelated feedback mechanisms in the model and plausible constraints governing its behaviour.

In the second stage of the model analysis, we used the results of the sensitivity analysis to pick suitable values for all parameters in the model for generic scenarios that represent interventions in the system at different levels (Table 1). For these intervention scenarios, we focused on the *personal preference* parameter for micro level, *tech and policy for plants* at the meso level (a variable intended to represent policy support or technological innovations for

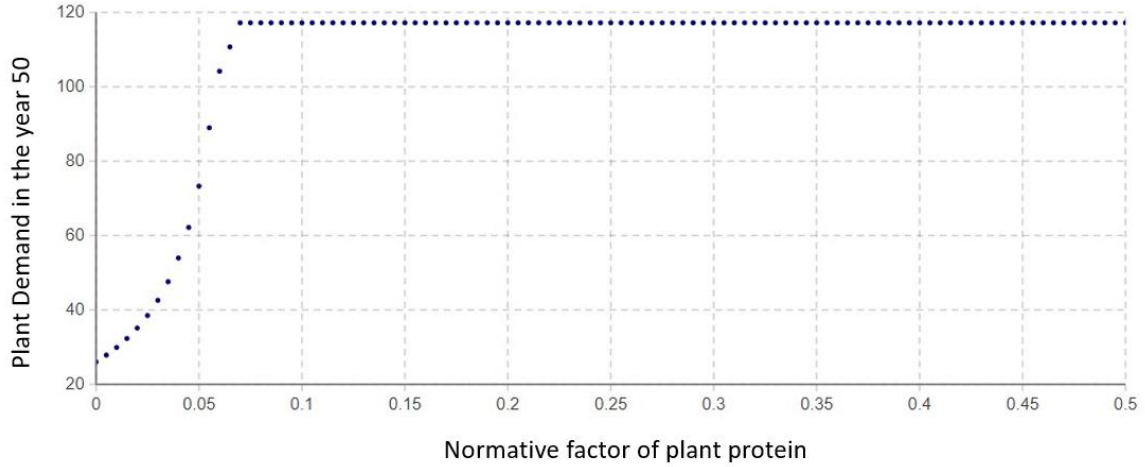


Figure 7: Response curve with y-axis showing plant-based protein demand at timestep 50 in the model, and x-axis showing different values of the normative factor of plant protein.

each protein type), and *consideration for plant* at the macro level (a variable representing preferences of landowners and farmers). Varying these parameters allows us to consider how interventions at different levels might interact. For example, to consider what might happen if policy interventions increased personal preferences for plant-based protein, but did not target the market or land use, or vice versa, or what might happen if we do all three? We do not consider whether interventions at these levels might be effective, but rather, assuming they will be effective in changing these parameters, seek to understand how they affect model behaviour or not. Also, note that we do not consider scenario uncertainty in this context, which is recommended when models are used for policy applications but not essential here for our purposes. An overview of the scenarios with their parameter settings listed, is given in Table 1. The results of the scenarios are given in Figures 8 and 9.

Table 1: The parameter values for nine intervention scenarios defined to run the model.

Scenario	Personal preference	Tech policy plant based	Consideration plant based
No intervention	Low: 0.2	Low: 0.1	Low: 0.05
Micro only (i.e., individual level)	High: 0.4	Low: 0.1	Low: 0.05
Meso only (i.e., market level)	Low: 0.2	High: 5	Low: 0.05
Macro only (i.e., land use level)	Low: 0.2	Low: 0.1	High: 0.5
Micro + meso	High: 0.4	High: 5	Low: 0.05
Micro + macro	High: 0.4	Low: 0.1	High: 0.5
Meso+ macro	Low: 0.2	High: 5	High: 0.5
All: High intervention in all levels	High: 0.4	High: 5	High: 0.5
Moderate intervention in all levels	Medium: 0.3	Medium: 2.5	Medium: 0.25

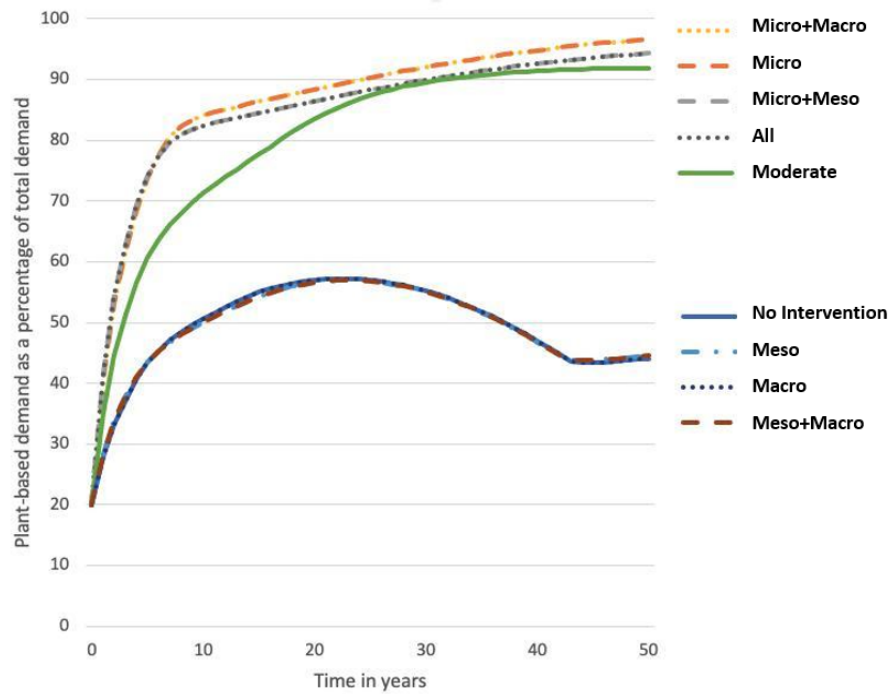


Figure 8: Plant-based demand as a percentage of total demand for scale-based scenarios detailed in Table 1.

Figure 8 shows plant-based protein demand as a percentage of the total protein demand across nine different scenarios outlined in Table 1. The "baseline no intervention scenario" illustrates a gradual increase in plant-based protein demand, which eventually stabilizes within the range of 50% to 60% of the total protein demand. However, it then experiences a decline before reaching a stable level towards the end of the simulation. Similarly, scenarios without any micro-level interventions mirror the behaviour of the "no intervention" scenario. Plant-based protein demand initially grows but subsequently decreases. In contrast, scenarios featuring interventions exhibit distinct patterns. In scenarios with micro interventions and those involving interventions at all levels with moderate values, plant-based protein demand surges to high percentages, exceeding 90% of the total demand. The "moderate scenario" displays a slower initial growth in plant-based demand, but it catches up with the other intervention scenarios over time.

These results suggest that within this model, the pivotal factor for achieving substantial and sustained increases in plant-based protein demand is the implementation of interventions at the micro level. These micro-level interventions show a significant and lasting impact on driving up the demand for plant-based proteins. Interventions at levels beyond the micro level have limited, if any, independent impact on increasing plant-based protein demand. Their influence becomes somewhat more pronounced when coupled with micro-level interventions, but remains relatively small compared to the substantial and lasting impact achieved by micro-level interventions alone.

The values for market capacity in plant-based proteins follow similar paths to demand, however, land use for different protein types does differ, as shown in Figure 9. Here, we can see with several scenarios (*all*, *micro + macro*, *moderate*, *micro + meso*, and *micro*) reaching around 70% of land use for plant-based protein production. Two scenarios (*Meso + macro*, and *Macro*) show land use for plant-based proteins rising, but then falling to around 40-50% of total land. Finally, the *no intervention* and *meso only* scenarios show land use for plant-based proteins collapsing halfway through the simulation. This further reinforces the finding that micro level intervention is key, but also suggests intervention at the land use level could affect behaviour, since only the scenarios with no intervention at micro and macro level show this collapse in plant-based protein land use.

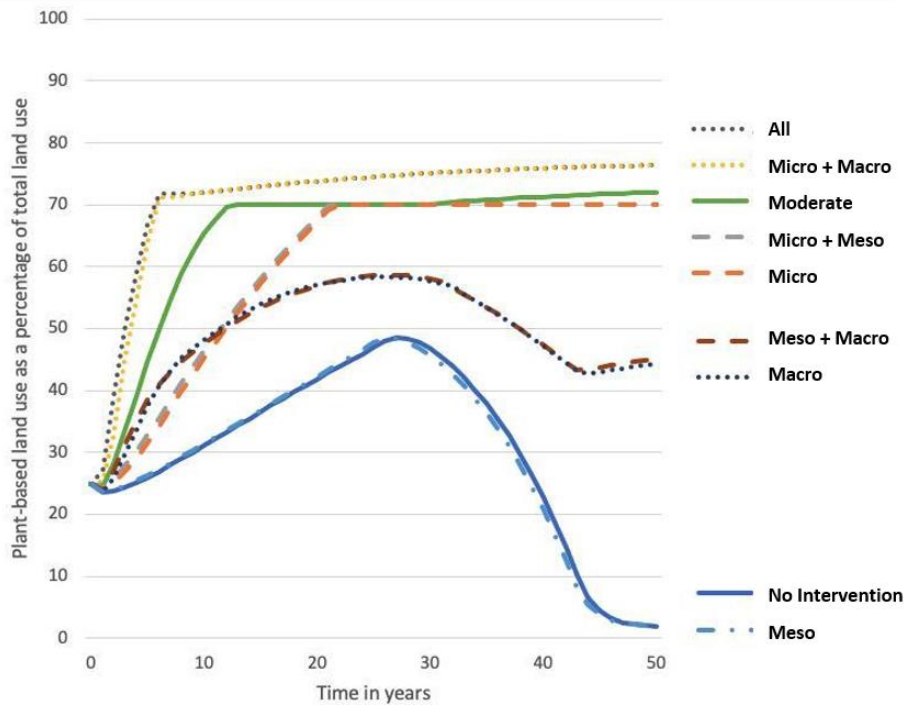


Figure 9: Plant-based land use as a percentage of total land use for scale-based scenarios

3.3 Diet model reflections

This model operates across three distinct levels: individual protein demand (micro scale), market capacity (meso scale), and global land use (macro scale). Utilizing a System Dynamics approach, it incorporates intricate feedback loops. The model primarily focuses on delineating these levels and establishing interconnections. Scenarios are deliberately designed to represent interventions at one or a combination of these levels, with the assumption that these interventions impact relevant variables. In the subsequent analysis, it is evident that micro-level interventions are pivotal, resulting in significant and lasting shifts in model behaviour. Conversely, interventions at the meso level exhibit a comparatively minor impact. This case study underscores the importance of comprehending cross-scale feedbacks and emphasizes that interventions at various levels play varying roles in shaping system behaviour.

Like any other model, it is subject to the influence of various human factors, such as intuition, heuristics, biases, and behavioural contexts. We have taken proactive measures to address these potential influences during the modelling process. This includes maintaining awareness of these factors, engaging in discussions among a group of modellers to deliberate on choices, and fostering a reflective approach to enhance the quality of our final inferences (Moallemi et al., 2020).

4. Case 3: A model of pyric herbivory in North American rangelands

Our third example describes a simple deterministic SES model representing pyric herbivory (the combination of fire and grazing) on a hypothetical cattle ranch in the rangelands of the southern Great Plains of North America. Rangelands cover approximately one third of the earth's land area, with at least one billion people dependent on these lands for their livelihoods (Follett & Reed, 2010; Ragab & Prudhomme, 2002). Most of the world's rangelands have been degraded by inappropriate land use practices (Millennium Ecosystem Assessment, 2005), primarily overgrazing by livestock (Milchunas & Lauenroth, 1993; Oesterheld et al., 1992). Overgrazing coupled with

suppression of fire, exacerbated by global changes in atmospheric CO₂, temperature, and rainfall, have facilitated continued encroachment of woody plants in what formerly were more open grasslands.

Research suggests that proper management of the combination of fire and grazing (i.e., pyric herbivory) at the local level can mitigate woody plant encroachment. Pyric herbivory connotes herbivory driven by fire such as to create a shifting mosaic of out-of-phase landscape patches (newly burned, recently burned with regrowth of forage plants, and not recently burned with woody plant encroachment) (for excellent conceptual overviews of pyric herbivory see Wilcox et al., 2018a, 2018b, 2021).

Today, ranch management decisions related to fire (use of prescribed fire) and grazing (adjustment of stocking rates) depend not only on local biophysical conditions but also are influenced by regional social and national political pressures. Rangelands are complex adaptive socio-ecological systems (Walker & Abel, 2001; Wang et al., 2020). Degradation of the ecological condition of rangelands (i.e., the ability to sustain livestock production and ranching livelihoods as well as provide ecological services such as filtering water, preventing of soil erosion, and sustaining biodiversity) has created social and political pressures to reduce stocking rates and to allow increased use of prescribed fire (Garmestani, 2013; Twidwell et al., 2016; Twidwell et al., 2019). Reduction of stocking rates decreases grazing pressure and use of prescribed fire reduces woody vegetation (“brush cover”), both of which increase the ecological condition of rangelands.

Here, we describe a simple deterministic SES model that represents the dynamics of pyric herbivory within the context of a hypothetical cattle ranching operation in the southern Great Plains of North America. The overall goal of the model is to demonstrate how small-scale changes in system components might set off a series of tipping points that could cascade through the system resulting in a large-scale regime shift. More specific goals of the model are: (1) to establish plausible links among ecological, social, and political system components that create the potential for the existence of tipping points; (2) to identify conditions under which tipping points could cascade across system components resulting in a regime shift from rangeland to woodland; (3) to identify interventions that could prevent tipping point cascades leading to regime shifts; and (4) to identify interventions that could reverse a regime shift.

4.1 Pyric herbivory model design

We have conceptualized the main causal relationships represented in the pyric herbivory model in Figure 10. We describe these relationships in greater detail and present their quantitative representations in the model in the Supplementary Material C. The model should be programmable in any software supporting numerical simulations based on difference equations and logical statements. We programmed the model in NetLogo (Wilensky, 1999).

4.2 Pyric herbivory model analysis

4.2.1 Identification of conditions under which tipping points could cascade across system components

To identify conditions under which tipping points could cascade across ecological, social, and political components in our hypothetical pyric herbivory system, we simulated two scenarios over the course of 50 years. The first represents conditions that maintain a dynamic equilibrium between rangeland and woodland. The second represents conditions that cause a cascade of tipping points across system components resulting in a regime shift from rangeland to woodland, which can no longer be reversed via the use of prescribed burns and the adjustment of stocking rates.

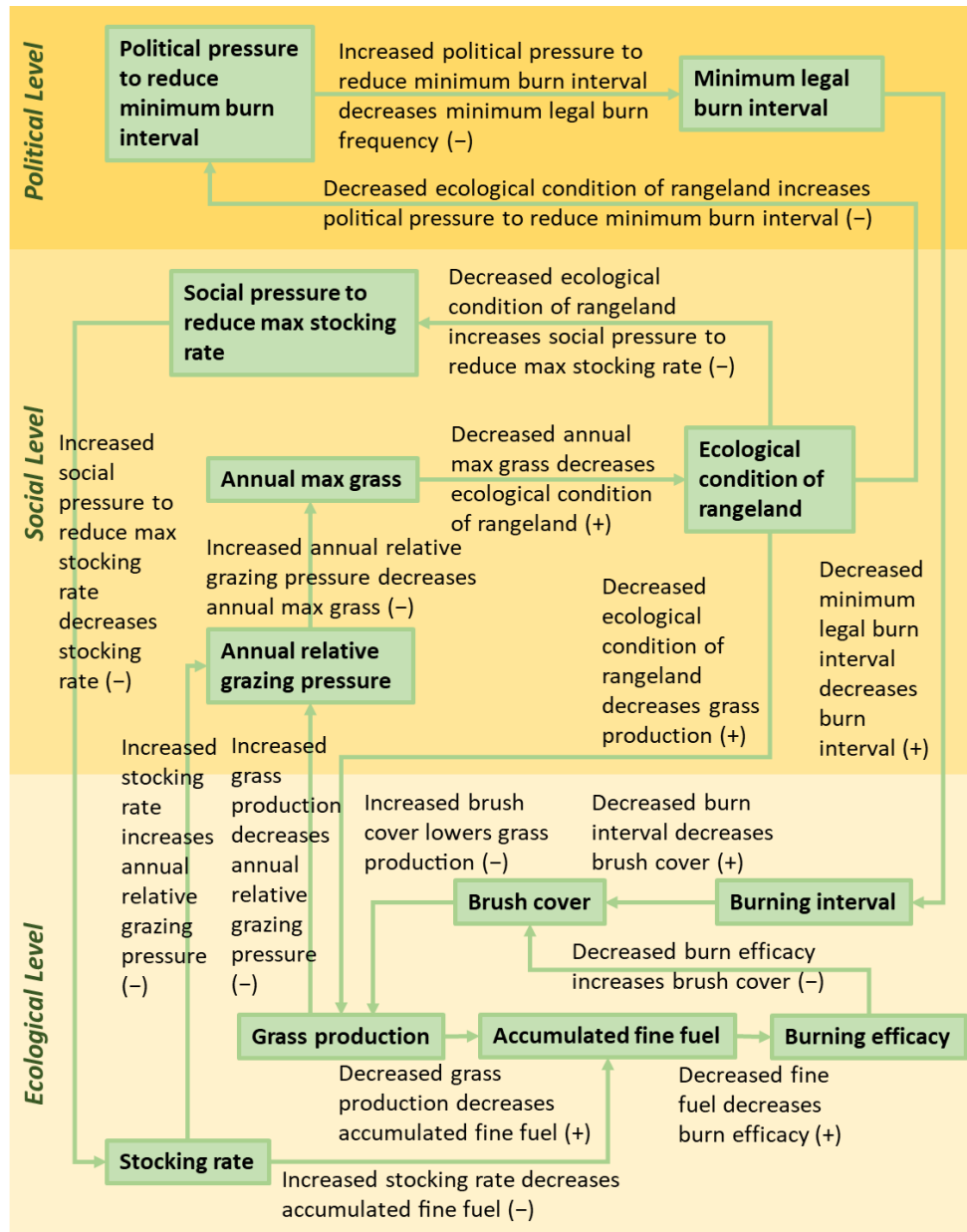


Figure 10: Conceptual model of the pyric herbivory system. See the Supplementary Material for details regarding relationships within and among levels. (+: varies in same direction; -: varies in opposite direction)

In Scenario 1 we assume an initial stocking rate of 5 cattle per unit area and an initial burning interval of 5-time units. We intend the former to represent a light stocking rate, or approximately 14 or 15 Animal Units per 100 ha (1 AU represents a cow with her calf) and the latter to represent the frequency of natural, lightning-caused fires, or approximately every 5 or 6 years. Under these conditions, sufficient fine fuel accumulates to allow efficient burns, which prevent brush encroachment beyond acceptable levels (Fig. 11a). Ecological condition declines slightly over a 50-year period (Fig. 11b), but not enough to cause enough social pressure to reduce stocking rate (Fig. 11c), and only enough political pressure to reduce the minimum legal burning interval by one year (Fig. 11d). Thus, we observe a dynamic equilibrium between rangeland and woodland.

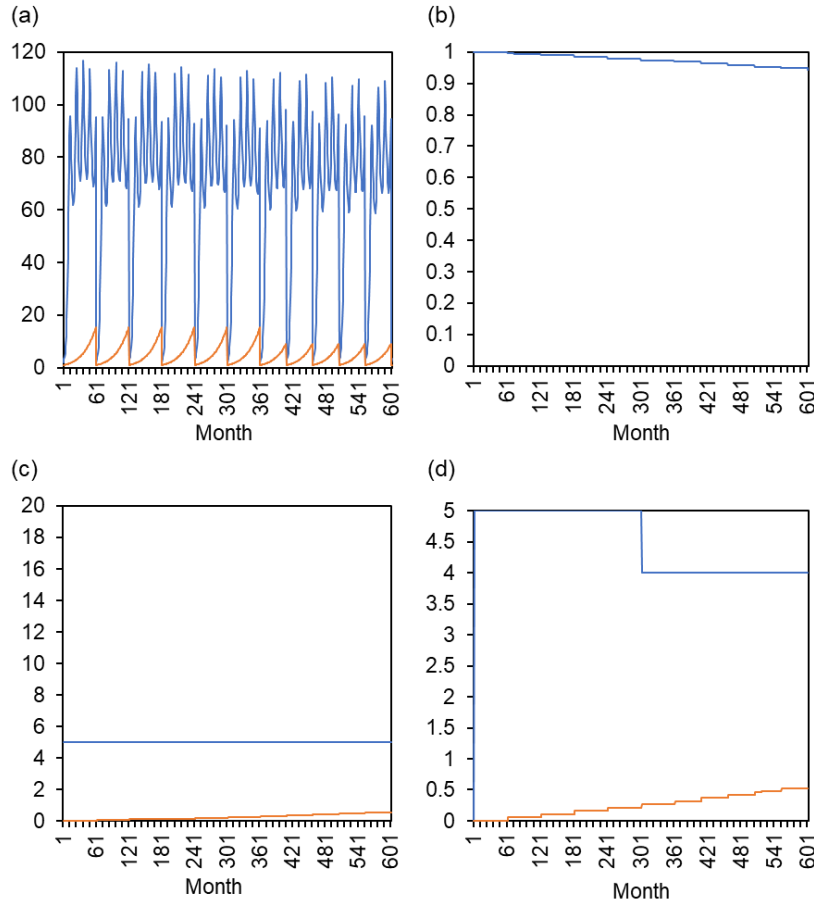


Figure 11: Simulation results for Scenario 1 indicating 50-year trends in (a) accumulated fine fuel (blue line) and brush cover (orange line), (b) ecological condition of the rangeland (blue line), (c) stocking rate (blue line) and social pressure (orange line) to reduce stocking rate, and (d) political pressure to reduce the minimum legal burning interval (orange line) and the minimum legal burning interval (blue line).

In Scenario 2, we again assume an initial burning interval of 5. However, we assume an initial stocking rate of 1, which is increased by 1 each year unless social pressure is strong enough to prevent it. Under these conditions, the increasing stocking rate leads to decreasing accumulations of fine fuel, which eventually results in inefficient burns that cannot reduce brush cover to acceptable levels (Fig. 12a). The combination of increasing stocking rate and increased brush cover leads to decreasing ecological condition (Fig. 12b), which increases social pressure enough to reduce stocking rate (Fig. 12c) and increases political pressure enough to reduce the minimum legal burning interval (Fig. 12d). However, the resulting tipping points in stocking rates (at approximately time step 265) and minimum legal burning intervals (approximately time step 240) occur too late to prevent the rapid phase transition from grassland to woodland (beginning at approximately time step 240), which could no longer be reversed via the use of prescribed burns and the adjustment of stocking rates.

4.2.2 Identification of interventions that could prevent tipping point cascades leading to regime shifts

We explored the ability of gradually intensifying social interventions to prevent tipping point cascades that cause a regime shift from rangeland to woodland. We simulated several versions of a third scenario in which we made the social pressure to reduce stocking rate increasingly responsive to declines in ecological conditions. More specifically, Scenario 3 consisted of a series of 10 simulations in which we sequentially changed the slope of the linear equation

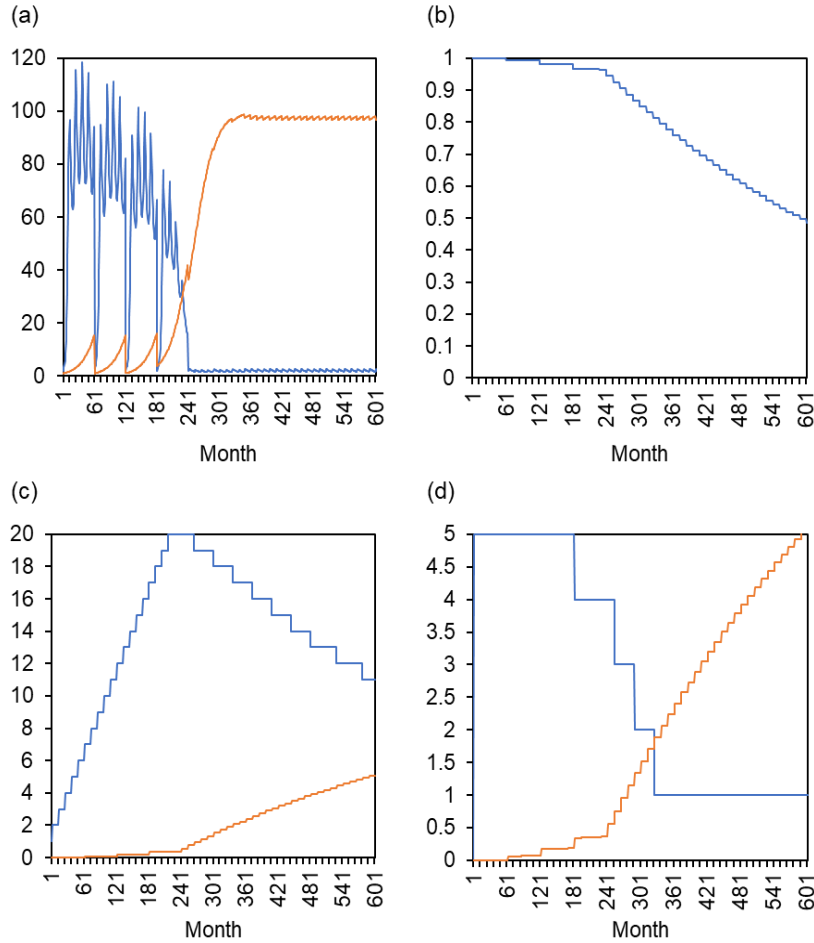


Figure 12: Simulation results for Scenario 2 indicating 50-year trends in (a) accumulated fine fuel (blue line) and brush cover (orange line), (b) ecological condition of the rangeland (blue line), (c) stocking rate (blue line) and social pressure to reduce stocking rate (orange line), and (d) political pressure to reduce the minimum legal burning interval (orange line) and the minimum legal burning interval (blue line).

relating maximum socially acceptable stocking rate to ecological condition (see Supplementary Material Table C1) from -10 to -1 in increments of 1. Otherwise, the model was parameterized as in Scenario 2.

In these simulations, as the social pressure to reduce stocking rate becomes more responsive to declines in ecological condition, the resulting tipping point in stocking rates eventually occurs early enough (at a slope value somewhere between -5 and -4) to prevent the rapid phase transition from grassland to woodland (Fig. 13). At a slope value of -5, the negative feedback from social pressure halts increases in stocking rates at a stocking rate of 11 (at approximately time step 110), but system dynamics still follow the trends exhibited in Scenario 2 (compare Fig. 12a, b, and c with Fig. 13, graphs in the left column). However, at a slope value of -4, social pressure halts increase in stocking rates at 9 (at approximately time step 85), and the tipping point cascade leading to a phase shift from rangeland to woodland is prevented (Fig. 13, graphs in right column). (Note: In these two Scenario 3 simulations, the initial change in the minimum legal burn interval due to political pressure (at approximately time step 240) is the same as in Scenario 2 (Fig. 12d)).

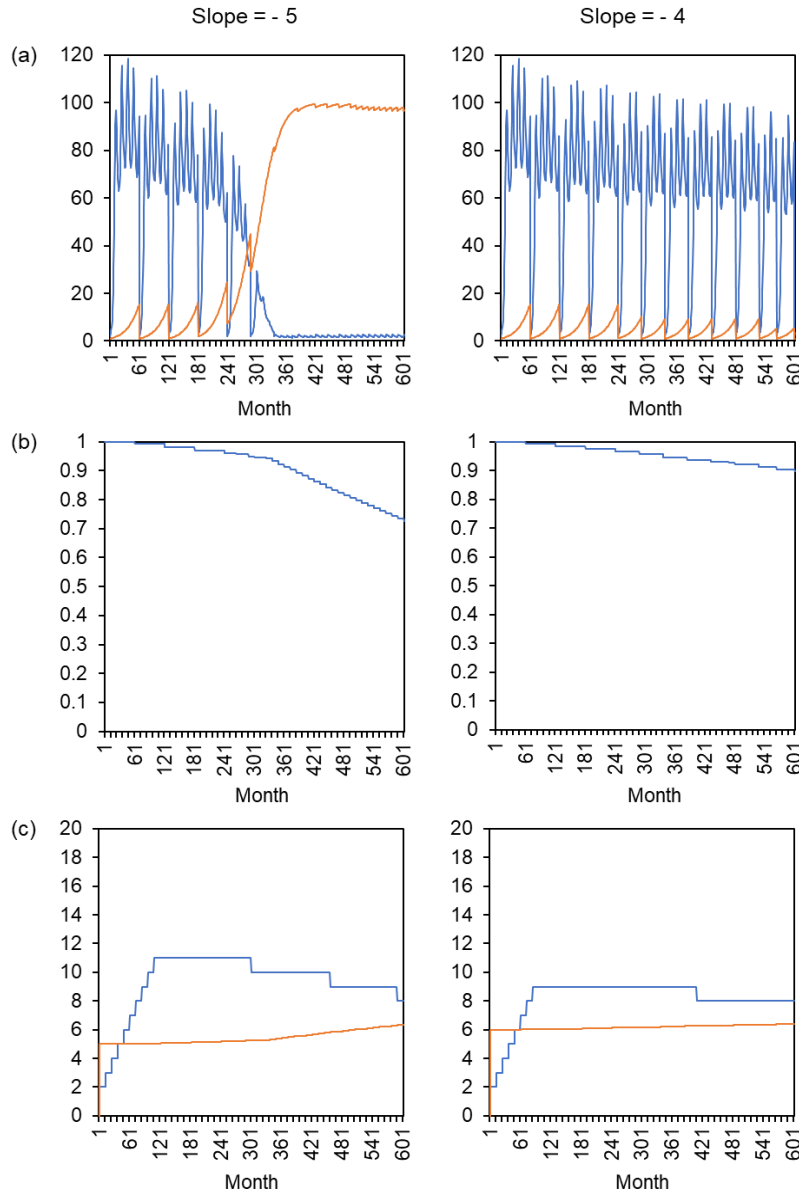


Figure 13: Simulation results for two versions of Scenario 3 in which the social pressure to reduce stocking rate was less (slope value = -5, left column of graphs) and more (slope value = -4, right column of graphs) sensitive to declines in ecological condition. Graphs indicate 50-year trends in (a) accumulated fine fuel (blue line) and brush cover (orange line), (b) ecological condition of the rangeland (blue line), and (c) social pressure to reduce stocking rate (orange line) and stocking rate (blue line). (See text for details).

4.2.3 Identification of interventions that could reverse a regime shift

We explored the ability of abrupt political interventions to reverse a regime shift from rangeland to woodland, that is, to restore rangeland conditions. We simulated a fourth scenario in which we assumed increasing political pressure to reduce the minimum legal burn interval, even after frequent burns no longer reduced brush cover, resulted in emergency legislation to restore the ecological condition of the rangeland. More specifically, in Scenario 4 (Fig. 14), when the index representing political pressure to reduce the minimum legal burn interval exceeded a threshold value of 2, all brush was removed (implicitly by mechanical or chemical means), the minimum legal burn interval was fixed at 5 (which maintained rangeland conditions in Scenario 1), the stocking rate was fixed at 1 (very low grazing pressure), and the ecological condition of the rangeland was increased annually by 0.01 while the stocking rate was

held at 1 (implicitly by natural processes). Subsequently, when the ecological conditions of the rangeland had increased to a level at which the index representing social pressure to reduce stocking rate was equal to 0, stocking rate was again allowed to increase annually by 1 until it reached the limit imposed by social pressure. Likewise, when ecological conditions had increased to a level at which the index representing political pressure to reduce the minimum legal burn interval was equal to 0, minimum legal burn interval was again allowed to respond to current political pressure. Otherwise, the model was parameterized as in Scenario 2.

In this simulation, political pressure to reduce the minimum legal burn interval exceeded the threshold value at approximately time step 340, well after the phase transition from grassland to woodland (beginning, as in Scenario 2, at approximately time step 240, see Fig. 13). The emergency reduction of brush cover resulted in a rapid increase in grass/fine fuel production and initiated a slow restoration of the ecological condition of the rangeland (Fig. 14). Thus, the regime shift was reversed. However, when the ecological condition had been restored (ecological condition = 1, at approximately time step 540) and the social and political pressures had dissipated (social pressure = 0, political pressure = 0), stocking rate was again allowed to increase annually by 1 until it reached the limit imposed by social pressure and minimum legal burn interval was again allowed to respond to current political pressure. Ecological conditions began to decline, and social and political pressures began to increase. Subsequently, a second regime shift from rangeland to woodland occurred (at approximately time step 800), which was followed by a second round of emergency legislation (at approximately time step 870) that again reversed the regime shift. A third regime shift from rangeland to woodland (at approximately time step 1340) was followed by a third round of emergency legislation (at approximately time step 1400), and the cycle continued.

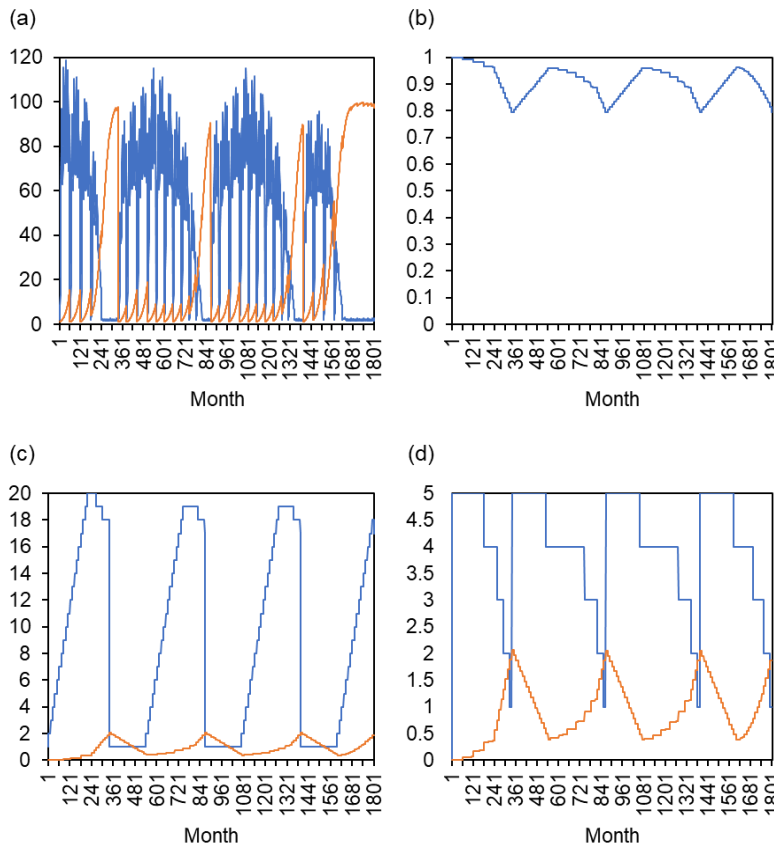


Figure 14: Simulation results for Scenario 4 in which increasing political pressure to reduce the minimum legal burn interval periodically resulted in emergency legislation to restore the ecological condition of the rangeland. Graphs indicate 150-year trends in (a) accumulated fine fuel (blue line) and brush cover (orange line), (b) ecological condition of the rangeland (blue line), (c) social pressure to reduce stocking rate (orange line) and stocking rate (blue line), and (d) political pressure to reduce the minimum legal burn interval (orange line) and minimum legal burn interval (blue line).

4.3 Pyric herbivory model reflections

Reflecting on the Case 3 results in a broader context, the importance of spatiotemporal interactions between fire and grazing in generating the dynamics of rangeland ecosystems serves to exemplify the pervasive importance of spatiotemporal interactions in generating the dynamics of coupled human and natural systems. Appropriately matched spatiotemporal interactions between fire and grazing (as in Scenario 1) maintain the shifting mosaic of different habitat patches across the landscape that characterizes sustainable rangeland ecosystems. Mismatched spatiotemporal interactions between fire and grazing (as in Scenario 2) can lead to an abrupt phase shift from grassland to woodland. Within the broader context of coupled human and natural (i.e., socio-ecological) systems, a mismatch between the spatiotemporal scales at which human decisions are made and the spatiotemporal scales at which the affected ecological processes function can lead to unintended consequences. For example, infrequently reviewed national environmental regulations that make sense for the average location during an average year can have negative impacts on the functioning of some local ecosystems, even during normal years, and on many local ecosystems during unusual years.

Absent from the Case 3 results, perhaps conspicuously so, is any mention of sensitivity or uncertainty analysis. This was by design, to simplify the presentation of results and to maintain our focus on modelling cross-scale feedbacks and tipping points in SES. Although beyond the scope of the present paper, we could conduct sensitivity and uncertainty analyses on our hypothetical cattle ranch model, assuming some degree of environmental and parametric uncertainty. And there almost surely would be parameter combinations that would not produce tipping points that cascade across scales. The impact of uncertainty (environmental, parametric, and that related to model structure) on the identification of cross-scale feedbacks and tipping points in aggregated SES models merits more attention within the broader field of forecasting uncertainty assessment.

5. Discussion

This paper has presented three examples of aggregate models of SES with cross-scale feedbacks and tipping points explicitly in mind. These cases move from abstract to real-world SES and integrate social and ecological elements in increasingly complex ways. As this occurs, the models, although all aggregate models, become more localised, and the feedbacks, alternative stable states, and tipping points become clearer and more intuitive, more related to the real-world. To address our goal of exploring how cross-scale considerations can be better brought into models of this type, we now consider the four questions we introduced above which revolve around model design, behaviour, interventions and analysis. Summaries of each case studies approach are described in Table 2.

5.1 The role of scale in model design

A range of themes emerge on each question. On model design, the case studies demonstrate how ‘designing-in’ cross-scale feedbacks and tipping points can come from one of two places. First, modellers can attempt to capture realistic representations of the system’s causal mechanisms and empirical regularities (as in the pyric herbivory example). Or, second, modellers can use a more intentional cross-scale model design, where selection of variables and their influence on the model is entirely framed around the cross-scale structure (which may be designed before the model itself) and purpose of the model. Depending on the model purpose, both of these approaches are likely to be valid approaches, and could deliver the same model design in theory, but in practice are likely to lead to different model designs or frame the analysis in different ways. For example, using an intentional cross-scale approach is likely to lead to a conceptually neat scale-based structure, as in the diet model, whereas a more empirical approach will lead to a less clear structure from a scale perspective, as in the pyric herbivory example, but one which may be desirable in other ways (e.g., more familiar to domain experts, or aligning with other analysis). Similarly, analysis could be focussed specifically around scale-based scenarios, or more closely tied to real policy scenarios. Modellers seeking to take a more explicit cross-scale approach to aggregate models will need to decide which of these two approaches they will take, i.e., build a cross-scale structure and design the model within this, or build a model based directly on the system structure, and then fit in a cross-scale framing around this.

Table 2: Overview of case study approaches to cross-scale questions.

Case study	Design question: How are cross-scale feedbacks and tipping points represented?	Model behaviour question: Under what conditions might we get cascading, upward-, or downward-, scaling tipping points in this system?	Intervention question: What interventions could be made to promote desirable cascades or scaling, or which might prevent unwanted ones?	Analysis question: How was the analysis of the model focused on understanding the cross-scale issues?
Case 1.1: Threshold model	Feedbacks are represented by bilateral law of mass action-type interactions, in which the rate of change depends on the densities of the involved variables. Tipping points result from bistability occurring in these feedbacks.	The initial densities of all involved variables should be large enough to prime the system for cascading towards a positive end state.	The densities of all variables should be increased to levels that exceed the bistability thresholds, so that the system can evolve to the desired positive end state.	The model is a set of differential equations and is analysed by steady state analysis and phase portrait analysis, in combination with simulations.
Case 1.2: Switch model	As in Case 1.1. In addition, a third variable describes a pulse to jumpstart the cascade.	The cascade towards a positive end state occurs if 1/ the process rates are within required ranges, 2/ the densities of the system variables are large enough, and 3/ the pulse variable is large enough and timed correctly.	If possible, adapt the process rates to such values that they accommodate the existence of an alternative positive end state. The pulse variable should be timed well and be made large enough to jumpstart the cascade.	The model is a set of differential equations and is analysed by a phase portrait and bifurcation analysis, in combination with simulations.
Case 2: Diet model	Identifying the levels was a core first stage of the modelling exercise. Once individual preferences, market production, and global land use were settled as the levels, all stocks and variables were intentionally framed around what level they belonged to. Connections between levels were then considered and sought out.	Interventions which affect the lowest, or most local scale, appear vital. Only in model runs where scenarios included effects at local scale, we see large and lasting shifts in model behaviour.	Interventions at the local level. Combinations with interventions at the macro level appear useful too, but meso level interventions less so.	Scenarios were specifically designed around interventions at one or a combination of levels. In this way, the analysis is specifically framed around interventions at different levels. The model does not consider efficacy of interventions, but rather assumes they have impacts on variable(s) at the appropriate level.

(Table continued on next page)

Table 2 (continued)

Case study	Design question: How are cross-scale feedbacks and tipping points represented?	Model behaviour question: Under what conditions might we get cascading, upward-, or downward-, scaling tipping points in this system?	Intervention question: What interventions could be made to promote desirable cascades or scaling, or which might prevent unwanted ones?	Analysis question: How was the analysis of the model focused on understanding the cross-scale issues?
Case 3: Pyric herbivory model	<p>Cross-scale feedbacks are represented via (a) the effect of changes in the minimum legal burn interval (scale level 3) on the burning interval (scale level 1) and (b) the effect of changes in the social pressure to reduce the maximum stocking rate (scale level 2) on the stocking rate (scale level 1).</p> <p>Key decisions involved the categorization of the various system components into different scale levels. In particular, the assignment of the ecological condition of the rangeland into the social component of the model (scale level 2) is important as opposed to the ecological component of the model (scale level 1).</p>	<p>Cross-scale cascades occur via increases in stocking rate (scale level 1), which lead (a) to decreased ecological condition of the rangeland and increased social pressure to reduce stocking rate (scale level 2) and (b) to increased political pressure (via decreased ecological condition of the rangeland) to reduce the maximum legal burn interval (scale level 3). Increased social and political pressure (scale levels 2 and 3) lead to decreased stocking rates and decreased burning intervals (scale level 1).</p> <p>The cascades described above are self-generated by the relationships in the model and therefore were easy to produce. The “tipping point” regime shift from rangeland to woodland was easy to generate via adjusting the slope of the linear equation relating maximum socially acceptable stocking rate to ecological condition.</p>	<p>An intervention that could prevent the regime shift from rangeland to woodland, which occurs in the baseline scenario, would be to make (“intervene” to make) social pressure to reduce stocking rate increasingly responsive to declines in ecological condition of the rangeland (via changes in the slope of the linear equation relating maximum socially acceptable stocking rate to ecological condition).</p>	<p>We explored various intervention scenarios (via changes in the slope of the linear equation relating maximum socially acceptable stocking rate to ecological condition).</p> <p>The analysis was focused on the cascading cross-scale effects of changes in the stocking rate via the structuring of the model such as to explicitly represent ecological, social, and political model components (sub-models, scale levels).</p>

5.2 Different ways of analysing interventions in cross-scale models

Themes around the intervention and analysis questions are more interrelated. In the pyric herbivory example, we see the ability of the model to examine relatively specific policy ideas which can reverse model behaviour, whereas in the diet model example, the policy scenarios are more stylised. However, the interventions identified are not really of different types because of the model design or the system itself (i.e., we could identify and model more specifics in the food system), but rather emerge from the type of analysis run. Analysis can be focussed on exploring model behaviour space (as in the abstract example), or on running more intuitive policy scenarios (as in the diet model example), or on both (as in the pyric herbivory example). This distinction is relevant for all models and applications but is likely of high importance when considering cross-scale behaviour. This is primarily because we are unlikely to be able to fully understand the behaviour of a model with many feedbacks and tipping points, unless we explore its behaviour space comprehensively.

Coarse-graining exploration of the model or using average results from a narrow set of runs is likely to miss important model results. Instead, a more thorough exploration of the model, with edge cases, and all scenarios and assumptions tested is needed (Iwanaga et al. 2021a). Using tools such as phase portraits appears a particularly appropriate and intuitive way of doing this (Van Voorn & Kooi, 2017) and that hence can provide useful information to understand the conditions under which cascading may occur.

However, this type of analysis is time consuming and technically challenging, and moreover, often not of immediate value to managers, stakeholders, and decision makers in a system. These types of model users value analysis and findings framed around the specific and feasible policy interventions, as opposed to full system behaviour (Barbrook-Johnson et al., 2023). Modellers will need to deal with the fact that different disciplines tend to be used to engaging with different types of model analysis (Wang & Grant, 2021). Mathematicians often aim for a more thorough analysis using tools like phase portraits and bifurcation analysis. Ecologists often use simulations and policy makers are used to dealing with scenarios. There will be trade-offs in the analysis conducted and the ways it is presented, which modellers using a cross-scale approach will need to negotiate.

5.3 Subjectivity and trade-offs in the modelling process

A set of broader themes emerge when we reflect more generally on these examples and the modelling process we undertook to develop them. The examples shed light on how subjective the decision of classing variables in different levels/scales can be, and reminds us that in aggregate models, all levels are 'treated' in exactly the same way by the model. In this sense, the scales we represent in the model(s) may be real in the real-world, but on the model's own terms they are constructs (Wang et al., 2023). That is, their meaning is created and used by us as modellers, not by the model in its operation.

The abstract example makes the subjectivity clear through the simple fact that the levels are just labels and have no meaning in relation to a real phenomenon. Moreover, finding real world relationships which follow those in the model, let alone real-world cross-scale relationships for the targeted SES, will likely be a conceptual and empirical challenge. This case study was relatively straightforward, requiring little iteration or development past the initial idea, mainly due to the use of existing, well-established models and a frame of analysis (i.e., phase portraits and bifurcation analysis). In the threshold model, the tipping point was explicitly inserted as mathematical formulation for demonstrative purposes, while in the other example (the switch model) the tipping point and cascading path emerged from the positive feedback interactions between the modelled variables. This is a choice to consider when modelling options for cascading in SES.

The diet model helps us with subjectivity, in the sense that stylised but intuitive relationships are clearly identified. However, they still sit in an awkward middle ground between being more plausible (judged by us as researchers), but because of the strong framing and influence of the cross-scale dynamics, still somewhat divorced from any structural or empirical reality. This 'uncanny valley' or sorts, of trade-offs between realism and abstract models, also affected our modelling processes. This was felt primarily in the number of iterations and depth of thinking required in different cases. Many iterations of the diet model development were needed. This was mainly due to the difficulty in designing the model with such a strong a priori scale framing, and with a multitude of processes to consider for inclusion or exclusion. Fitting plausible mechanisms into a neat scale structure was the challenging aspect (readers can be the judge of whether we did this well or not). Once this

was in place, developing and implementing the analysis was straightforward from a scale perspective; since the structure was in place to easily identify and implement scenarios for analysis.

The pyric herbivory case demonstrates the inverse point, where the relationships are more solidly based on real world behaviour, but it becomes much less clear in which level we should class a variable, or indeed if there are conceptually neat levels or scales in the model. For example, here we have 'social' and 'political' levels which are intuitive, but the line between them is fuzzy, and ultimately, subjective. While the pyric herbivory was relatively simple to design, being based on a more realistic structure and using existing models as inspiration, designing the analysis to be scale-based was more iterative here. At first, it was focussed on understanding the model behaviour space, and only moved into assessing scenarios as a second qualitatively different stage of analysis (Iwanaga et al. 2021b).

6. Conclusion

This paper has presented and reflected on three case studies of aggregate models being used to explore cross-scale feedbacks and tipping points in SES. Our aim in doing so has been to support the better representation of cross-scale dynamics in SES and contribute to the growing demand for more cross-scale modelling of SES (Lippe et al., 2019). From these case studies we see some emerging themes and lessons across model design, model behaviour, insights on policy interventions, and analysis of models. Most salient of these for modellers looking to adopt a cross-scale approach in aggregate models are:

1. Being clear about whether we are starting with a cross-scale framing for a system and modelling within it, or applying a framing to an existing or more empirically or structurally-driven model.
2. Understanding what is driving cross-scale behaviour in our model and the SES of interest.
3. Managing trade-offs between exploring the full model behaviour space (which may be large and complex given cross-scale feedbacks) and providing relevant and usable policy analysis for model users and decision makers (and in doing so, what modes to use to present model results).
4. Emphasizing the subjective nature of introducing scale to aggregate models, and how this can shape the modelling process in different ways - sometimes making model development more difficult, other times making analysis more difficult.

Taken together with the other papers in this special issue, it is clear the time for theory and position pieces calling for cross-scale modelling is done. We now need more practical tools for modelling cross-scale dynamics, more detailed and high-quality examples, and more reflections and lessons on what works. Another missing piece is the use of these ideas and methods in applied modelling used in decision making. Whether it be modelling of the climate tipping points and their interaction with the economy in integrated assessment models, large models of land use change, or other SES topics, there is a clear absence of these highly influential models using advanced representations of scale, feedbacks, and tipping points. This should be an urgent area for development of these models.

CRediT statement

PBJ - conceptualisation, methodology, validation, writing - original draft, writing - review and editing, visualisation, supervision, project administration, funding acquisition. GvV - conceptualisation, methodology, software, validation, formal analysis, writing - original draft, writing - review and editing, visualisation. HHW - conceptualisation, methodology, Software, validation, formal analysis, writing - original draft, writing - review and editing, visualisation. FZ - conceptualisation, methodology, Software, validation, formal analysis, writing - original draft, writing - review and editing, visualisation. WEG - conceptualisation, methodology, Software, validation, formal analysis, writing - original draft, writing - review and editing, visualisation. ZP - methodology, validation, writing - original draft, writing - review and editing, visualisation. ML - conceptualisation, writing - original draft, writing - review and editing, visualisation, project administration, funding acquisition.

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Supplementary Material

The Supplementary Material can be found online at: <https://sesmo.org/article/view/18616/18210>.

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