Robust strategies for forest wildfire mitigation under uncertainty

Moritz Hildemann[1*], Amelia B. Pludow[2], Alan T. Murray[2], Edzer Pebesma[1] and Judith A. Verstegen[3]
[1] Institute for Geoinformatics, University of Münster, Germany
[2] Department of Geography, University of California Santa Barbara, United States.
[3] Human Geography and Spatial Planning, Utrecht University, The Netherlands.

Abstract

Modern forest management is often focused on minimizing wildfire risk by regaining fire-resiliency from historical conditions. This process is labor and resource intensive and it is usually infeasible to treat entire forests. Therefore, forest treatment allocation optimization is applied to help identify the best mitigation plan, with a key objective being the minimization of departure from historical landscape conditions. However, the vagueness and imprecision of historical landscape condition models and their corresponding departure indicators due to data limitations often necessitate the use of simulated substitutes. Since spatial optimization, integral for identifying optimal mitigation plans, relies on simulation outputs, addressing uncertainty becomes a crucial concern. We analyze optimization outputs for different historical condition scenarios and propose strategies to identify robust alternatives in two steps. First, we conduct a series of spatial optimizations using a weighted-sum method, systematically varying the importance assigned to different objectives for each scenario of historical conditions. This process helps us understand how changes in the assumptions about past forest conditions and the relative weights assigned to each assumption affect the optimal management plans. Second, we analyze the resulting solutions to identify patterns and overlaps among the optimal spatial configurations across scenarios. By focusing on the identification of common patterns, we develop a strategy to select solutions that are robust to uncertainty in historical condition models. We find that the strategy based on identifying commonalities among optimal solutions yields management plans that are, on average, 8.9% more robust compared to plans derived without explicit consideration of robustness. We conclude that the commonality seeking strategy in solution space alleviates risk in decision making. These insights highlight the importance of developing strategies for applying robust strategies in spatially explicit optimization problems.

Keywords

forest fire mitigation; forest management; historical range and variation simulation; uncertain spatial data

1. Introduction

Spatial optimization models often utilize simulation outputs to evaluate solutions. Simulation model outputs exhibit uncertainties that can influence optimization results via uncertainty propagation. If the uncertainties of the simulation models are known and can be quantified, then the propagated uncertainty can be assessed (Zhang, 2021). However, uncertainties are not always quantifiable. Simulated events in the far future or past, for example, can often neither be validated nor can the uncertainty be quantified (Maier et al., 2016). In case of unquantifiable uncertainties, one option is to neglect uncertainty and use a single scenario that is constructed

Correspondence:
Contact M. Hildemann at moritzhildemann94@gmail.com

Cite this article as:
Hildemann, M., Pludow, B.A., Murray, A.T., Pebesma, E., & Verstegen, J.A.
Robust strategies for forest wildfire mitigation under uncertainty
Socio-Environmental Systems Modelling, vol. 7, 18725, 2025, doi:10.18174/sesmo.18725

Socio-Environmental Systems Modelling
An Open-Access Scholarly Journal
http://www.sesmo.org

with the available information. The alternative is to simulate and utilize multiple equally plausible scenarios for the future or the past (Lempert et al., 2003).

One spatial optimization application that depends on simulated events of the past due to scarce data is forest treatment allocation to mitigate wildfires. A modern strategy in wildfire mitigation is to regain historical landscape conditions, which are reported to have been fire resilient (Keane et al., 2009). The fact that historical conditions have not been documented imposes a challenge. Murray et al. (2022) overcome the problem by utilizing the most plausible scenario of past conditions to achieve the goal to optimally allocate forest treatment activities. Given the absence of evidence against the departure indicator, we accept the plausibility of the single scenario. What justification is there for not assuming its plausibility and working with it?

The main reason is that a single scenario or a single realization of modeled uncertainties does not allow quantification of robustness associated with solutions. In robust optimisation, a solution is considered robust if it maintains acceptable performance across all scenarios, particularly under worst-case conditions, rather than being optimal in any specific scenario (Gabrel et al., 2014). Another way of identifying robust solutions is to quantify sensitivity of objective values to small changes in various model parameters and select the least sensitive solution from a range of solutions (Deb & Gupta, 2005). In a research stream called deep uncertainty, solutions that are generally insensitive to uncertainty are considered robust (Lempert et al., 2003). All these approaches require a set of realizations or a set of scenarios to assess robustness of solutions.

Various strategies to identify robust solutions exist, with strategies tailored to the given problem (Shavazipour et al., 2021). Robust optimization, for example, relies solely on the objective values, i.e. the worst-case objective value from a distribution of uncertainties. Well-defined uncertainties, mostly aleatory uncertainties that follow a known distribution (Hassani et al., 2023), are required for robust optimization. This prerequisite is not given in historical landscape condition simulation models. In fact, it remains unresolved which variables are most appropriate for describing historical conditions (Keane et al., 2009). Alternative to the described robust optimization approach, other strategies focus on similarities between solutions and evaluate robustness based on design configurations, rather than solely on the objective space. In spatial optimization, leveraging spatial relations such as proximity, overlap, containment, and distribution patterns has been instrumental in identifying robust solutions. Studies by Dong et al. (2019), Christiansen et al. (2015) have successfully utilized these relations within design variables to produce robust outcomes, while Hildemann & Verstegen (2021) demonstrated that identified patterns within solutions serve as indicators of robustness.

This paper identifies two gaps in the current literature. First, research regarding strategies that extend solely objective value-based robustness quantification is still limited in spatial optimization. Second, robust decision-making strategies are lacking for spatially explicit allocation optimization models that rely on simulation model outputs that cannot be validated.

This work develops problem-specific strategies for robust decision-making in spatially explicit allocation optimization to overcome the identified gaps. We propose two strategies to identify robust solutions using three different simulation models to quantify departure from historical landscape condition. The first strategy utilizes existing multi-criteria decision-making approaches based on the objective space. The second strategy identifies relations within realizations of optimal spatial configurations under uncertainty.

We aim to answer the following research question: What insights for robust decision making can be gained with an objective space-oriented strategy and a solution space-based strategy in spatially explicit allocation optimization under uncertainty?

2. Background

2.1 Robust solutions

The term *robust solution* has been used in various contexts in optimization and decision making. In robust (or stochastic) optimization, robustness is generally achieved by considering the worst-case scenario realization for the evaluation (Gabrel et al., 2014). The evaluation is based upon the worst-case scenario in objective space, i.e. how favorable the objective value is if the worst case occurred. In other studies, a robust solution is defined differently, e.g. as the least sensitive solution to small changes in model parameters that can be quantified by the distribution of the objective values (Deb & Gupta, 2005). Moreover, Labbé et al. (1991), analyzed the sensitivity of a network-based location-allocation optimization to various input parameters by characterizing a tradeoff curve between the magnitude of perturbation of the input parameters and the optimality of solutions. A different yet underrepresented principle is to analyze decision variables (or design configurations) within solutions to determine robustness, not solely relying on the objective values to assess robustness. This approach has been, among other contexts, applied in network optimization. Dong et al. (2019) identified and removed network edges that have a higher probability to fail in earthquake scenarios. In the domain of Computer Science, the Hamming distance has been applied to identify robust patterns within uncertain scenarios (Grüne, 2024), which is a measure that quantifies the number of positions of equal length that differ between two strings.

It is assumed, that similar measures can be utilized to assess robust patterns in solutions to spatial optimization problems by measuring either the attributes or spatial relations of spatial objects. Based on such a similarity measure, emerging spatial relations within solutions from various realizations of modeled uncertain parameters can be identified. Such patterns can either be robust or highly varying components of solutions when compared to other solutions of similar quality. Hildemann and Verstegen (2021), for example, analyzed frequency patterns of land uses in optimal solutions and concluded that quantifying the emergence of patterns within multiple optimal solutions could serve as indicator for robustness in solutions or subset of solutions. In the methodologically related location-allocation spatial optimization under various data aggregation levels, the notion of *stable solutions* has been discussed detached from the context of uncertainty (Fotheringham et al., 1995). Here, a solution is considered stable if its quality is within a specified percentage deviation from the optimal solution of the disaggregate data (Murray & Gottsegen, 1997).

2.2 Historical forest conditions

The historical range of variability (HRV) aims to represent historical forest conditions before human interventions. These are then compared with current conditions for estimating the departure from historical conditions, which characterizes the capability of forests to persist wildfires and other short-time destructive forces (Albrich et al., 2020). The most common method to compute HRV is simulation (Keane et al., 2009) with the simulation models being highly diverse: the selected modeled variables differ, as well as their spatial and temporal scale and resolution (Keane et al., 2009). Moreover, the HRV differs per selected time frame due to changing climate conditions, and the simulation models also differ in the description of the landscape simulation outputs. For example, Nonaka and Spies (2005) describe the historical landscape with seven metrics (amount of forest, tree patch sizes, edge abundance, patch shape, diversity of forest age, patch connectivity, and patch contagion), whereas McGarigal and Marks (1995) use 59 metrics to describe the landscapes. Related to that, departure indicator quantifications vary. One quantification metric is Sorensen's Index, which measures aerial differences for a set of binned metrics from simulation intervals to the same binned metrics of present conditions (Keane et al., 2009). Different sets of metrics and different numbers of classes for binning the metrics result in different indicator values. Another departure quantification method is a regression-correlation significance measure (Steele et al., 2006). Bissonette and Storch (2007) propose using principal components analysis for measuring the departure from historical conditions. The different departure quantifications also result in different ranges of output departure values.

2.3 Strategies to identify robust solutions under uncertain historical landscape conditions

As described above, historical landscape condition simulation models are highly uncertain in many ways. Every decision regarding the modeled variables, simulation parameters, spatial and temporal resolution, time frame, and departure quantification may result in different departure indicator values.

Moreover, the probability distributions of the model parameters are unknown. The high number of possible combinations for defining the model, the lack of known probability distributions that describe historical landscape conditions, as well as the lack of ground truth data to validate or compare simulation models make it impossible to determine the most appropriate model to quantify departure from historical conditions. In such cases, Lempert et al. (2003) suggest using multiple plausible scenarios that are regarded as equally probable. According to Lempert et al. (2003), an exploratory scenario-based analysis can then be performed to identify robust solutions that perform adequately under multiple plausible scenarios (Maier et al., 2016). In the context of this study, it would help to answer the question *"what allocation patterns minimize the departure from historical conditions over a set of possible historical conditions irrespective of which of the pasts occurred?"*

Finding optimal solutions under uncertainty that are robust over a set of possible scenarios requires an appropriate robustness metric for the given problem (Shavazipour et al., 2021). Marchau et al. (2019) claim that no robustness criterion is best in all circumstances and that the metric must facilitate the framed problem. No such metric exists for the problem addressed in this study.

3. Methods overview

This study utilizes a scenario-based optimization approach in two phases to identify robust forest management solutions that perform well under a range of plausible historical conditions (see Fig. 1).

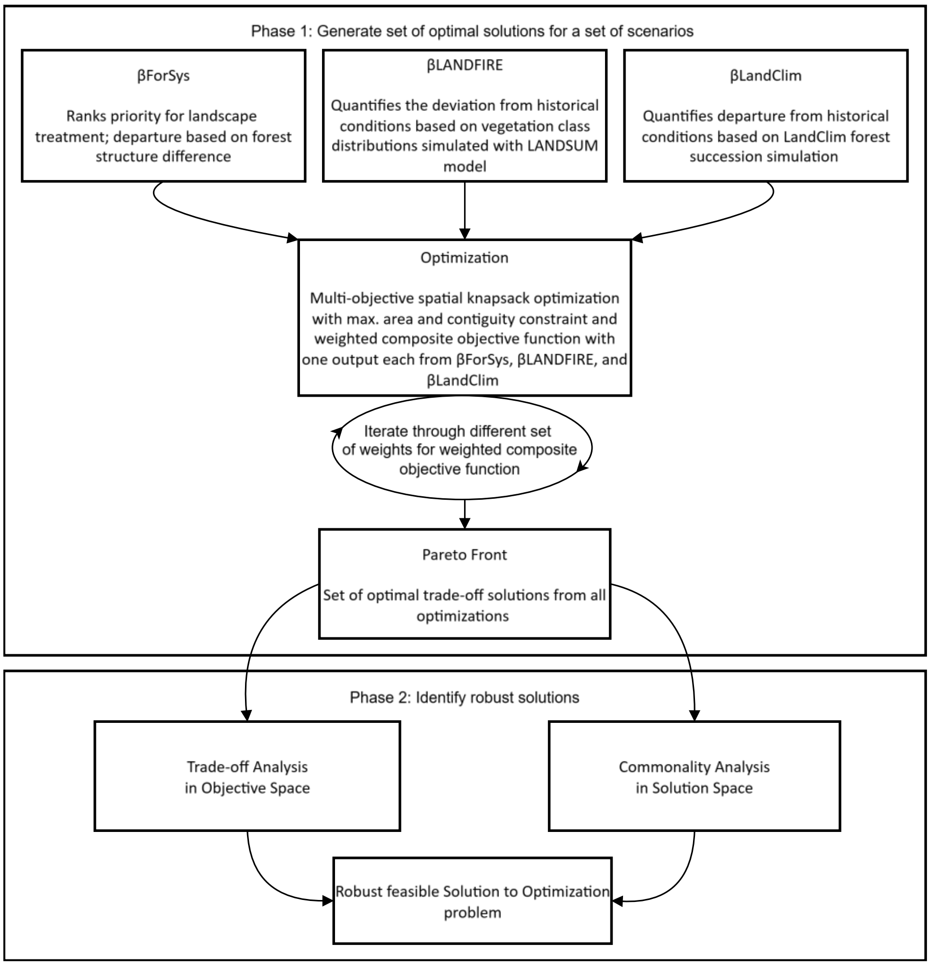

Figure 1: General flow of methods used in this paper to identify robust solutions to a deeply uncertain spatial optimization problem.

In the first phase, we define three distinct scenarios of historical forest conditions, each represented by a specific departure indicator: β_{ForSys}, $\beta_{LANDFIRE}$, and $\beta_{LandClim}$. $\beta_{LANDFIRE}$ expresses percent deviation of current vegetation from historical conditions, β_{ForSys} assigns a rank based on the departure of the forest structure, and $\beta_{LandClim}$ measures similarity to simulated historical conditions using a statistical index. These indicators derive from different data sources and modeling approaches but share the common objective of quantifying departures from historical forest conditions.

Each indicator feeds into a composite objective function within a multi-objective spatial optimization problem. This problem is addressed using a spatial knapsack model with a contiguity constraint, ensuring that the selected management areas are contiguous. By systematically varying the weights of each indicator in the composite objective function, we generate a set of Pareto-optimal solutions. The deliberate simplification of using one scenario per modelling approach simplifies the identification of common patterns and trade-offs within the set of Pareto-optimal solutions.

In the second phase, we analyse the generated set of Pareto-optimal solutions to evaluate how consistently these strategies perform across different scenarios to assess robustness. We employ two complementary strategies: trade-off analysis in the objective space to assess solution performance and clarify trade-offs under each scenario, and commonality analysis in solution space to identify areas consistently prioritized across scenario weightings. Importantly, this analysis ensures that robust solutions are not just frequent combinations of individual units, but also satisfy the contiguity constraint, producing management areas that are operationally feasible to implement in practice.

4. Identify Pareto optimal solutions for set of equally probable metrics

4.1 Forest treatment allocation optimization model

Forest treatment allocation optimization is based on the spatial knapsack problem formulated by Church and Murray (2009) with a contiguity constraint after Shirabe (2005). The spatial knapsack optimization including the contiguity constraint has been formalized in Murray et al. (2022) as follows:

1) $Maximize\ \sum_i \beta_i X_i$
2) $Subject\ to\ \sum_i a_i X_i \ \leq T$
3) $\sum_{j \in N_i} Y_{ij} - \sum_{j \in N_i} Y_{ji} \geq X_i - MV_i \forall i$
4) $\sum_i V_i = 1$
5) $\sum_{j \in N_i} Y_{ij} \leq (M-1) X_i \forall i$
6) $X_i = 0{,}1\ \forall i$
7) $V_i = 0{,}1\ \forall i$
8) $Y_{ij} \geq 0 \forall i, j$

where

i: index of land management units,
β_i : the benefit of treating unit *i*,
α_i : the area of unit *i*,
T: the maximum treatment area,
X_i : the decision variable indicating treatment (X_i = 1) or no treatment (X_i = 0) for unit X_i,
N_i : spatial neighbors of unit *i* (here based on adjacency),
M : number of management units,
V_i : the decision variable indicating whether or not unit *i* is a sink,
Y_{ij} : decision variable indicating flow between units *i* and *j*.

The optimization model is used to identify the best contiguous patch with *M* (one or several) treatment areas for maximizing the benefit β when treating the units belonging to the patch. In this case, β is the departure from historical conditions. Three departure indicators are used, referred to as β_{ForSys}, $\beta_{LANDFIRE}$, and $\beta_{LandClim}$. β_{ForSys} quantifies how much current vegetation conditions deviate from historical norms (HRV), ranging from 0% (no departure) to 100% (complete departure). β_{ForSys} is a unitless indicator (0–6.5) that ranks spatial units by their prioritization for fuel treatments for fire risk mitigation based on the departure of current to historical forest structure. $\beta_{LandClim}$ quantifies the similarity between simulated historical-range-of-variability (HRV) vegetation structure and current conditions using the Sørensen's index, ranging from 0 (maximum departure) to 1 (no departure).

The threshold *T* defines the maximum area treated (Eq. 2). The contiguity constraint ensures that selected land units inhibit "the quality of a single region being connected" (Shirabe, 2005). Selected land units (X_i) are represented as a connected fluid network with sinks (V_i), where each patch has a sink. The contiguous region is represented by the patch, where one unit serves as sink. The linear equations Eq. 3 - Eq. 8 define the conditions that need to be fulfilled for a region to be contiguous; the details of the network contiguity constraint are described in Shirabe (2005), and a pseudo-code for finding the contiguous regions can be found in Supplementary Material A.

The optimization problem is an Integer Linear Programming (IP) problem with linear constraints and objective functions (Nayak, 2020) for which exact solutions can be obtained.

4.2 Three indicators to quantify departure from historical conditions

The three different departure indicators serve as optimization coefficients to approximate the benefits β_{ForSys}, $\beta_{LANDFIRE}$, and $\beta_{LandClim}$ associated with different treatment configurations. The output from the departure simulation models is one departure indicator value per model per land unit; the ranges of values differ per indicator. The different ranges stem from multiple possibilities to quantify the departure from historical conditions (Section 2).

The LANDFIRE (2016) Vegetation Departure Indicator quantifies the deviation of current vegetation class distributions from the historical range of variability (HRV) for each area, as simulated with the LANDSUM model (Keane et al., 2006). HRV is defined following the Fire Regime Condition Class (FRCC) protocols by Hann et al. (2004). The indicator compares the relative proportions of different vegetation classes and structural stages observed today to those expected historically, as modeled by combining knowledge of fire regime influences, successional dynamics, and disturbance regimes. The result is $\beta_{LANDFIRE}$, quantified as a percentage value (0–100%), with higher values indicating greater departure from historical conditions.

The ForSys departure indicator β_{ForSys} is a quantitative metric used in spatial forest scenario planning, which was developed by the Social and Ecological Resilience Across the Landscape (SERAL) project. ForSys (Ager et al., 2019) ranks the priority for landscape fuel treatment interventions based on how much current forest structure derived from LiDAR data (McGaughey & Carson, 2003) deviates from reference ecological conditions. These reference conditions were simulated using forest landscape models that represent the expected range of structural variability under natural disturbance regimes and historical management. Higher values indicate high difference from reference conditions. Input metrics to quantify the deviation are the mean tree clump size, trees per acre and the proportion of gaps between trees.

A third departure indicator $\beta_{LandClim}$ is introduced based on HRV simulations using the forest succession simulation LandClim (Schumacher et al., 2004). This simulation applies the methods described in Bugmann (1996) but with study area-specific forest species and parameters. It is a landscape simulation model that is sensitive to various soil and climate conditions and includes disturbances from fire, wind throw, bark beetle outbreaks, and browsing. The inputs of the simulation are the elevation, aspect, slope, soil available water capacity, monthly climate, management activities, and land type specific parameters with different fire ignition coefficients, maximum vegetation biomass, browsing intensities etc. The key variables to describe the landscape conditions are the biomass in (t/ha), tree density (stems/ha), species composition, as well as additional sub-module specific outputs like burned area, areas affected by droughts, bark beetles, or wind throw. The following metrics are used to describe landscapes to compute the departure indicator: the simulated mean vegetation height and dominant vegetation class per cell and time interval are compared with the current vegetation height and the vegetation classes using the Sorensen's index (Keane et al., 2009) to quantify departure. The simulation is set up with random initial vegetation, a simulated time frame of 1000 years as used in Keane (2012) and Snell et al. (2017), and default values for fire, wind throw, bark beetle, and browsing parameters. The departure indicator ranges from 0 to 1. The details are described in Supplementary Material B.

4.3 Trade-off solutions from the weighted-sum method

A deterministic optimization is performed with the weighted sum method (Marler & Arora, 2010) for analyzing the scenario trade-offs with the different β-values. This method performs optimization with a composite objective function from multiple objectives. The composite objective function computes a composite benefit β with the three weighted (min-max-normalized) departure indicators serving as optimization coefficients that quantify benefits β_{ForSys}, $\beta_{LANDFIRE}$, and $\beta_{LandClim}$. All weight combinations must sum up to 1, the weights can range from 0 to 1, and the weight increment by 0.01. By repeatedly assigning different combinations of weights for each of the objective functions and performing the optimization, we obtain multiple optimal objective function weight combinations (Marler & Arora, 2010). The advantage is that the composite objective function remains linear, just as its component objective functions. The drawback associated with this method is that it only guarantees finding all optimal solutions when the Pareto front has a convex shape (Boyer et al., 2013), which is assumed in the given problem.

From all solutions obtained with the weighted sum method, those weight combinations are identified that lead to optimal trade-off objective values of the single objectives. The solutions associated with optimal weight constellations are then used for the trade-off analysis between the different departure indicators to identify robust solutions.

5. Post optimization robustness evaluation strategies

The challenge is to determine a scenario-based analysis approach and a robustness metric that facilitate the framed problem (Marchau et al., 2019). The primary goal is to provide decision-makers with insights about the robustness of solutions despite the lack of knowledge of what departure indicator best captures the departure from historical conditions. Here, two strategies are proposed to a problem specific scenario-based analysis. In the first strategy, trade-offs in the objective space are identified where each objective represents one of the simulation models (Section 4.3) and only the optimal trade-off solutions from the weighted sum method (Section 4.4) are presented. The second strategy identifies commonalities in solution space and objectively presents a single robust solution. Both strategies are forest treatment allocation-specific metrics but could also be applied with minor adaptations to identify robust solutions in other spatially explicit combinatorial allocation optimizations.

5.1 Identification of robust solutions using trade-offs in objective space

The optimal trade-off solutions obtained using the weighted sum method represent a Pareto set of alternatives, from which decision-makers must select a preferred option, which is a common task in multi-criteria decision analysis (Jankowski, 1995). To explore these trade-offs in objective space, a trade-off hyperplane is used to visualize the relationships between the optimal solutions (Sakawa & Yano, 1990). One effective method for visualizing these trade-offs is RadViz, a projection-based multivariate technique that arranges the values of the weighted composite departure indicators in a radial layout (Deb, 2001; Tusar & Filipic, 2015). The solution nearest the center of the RadViz plot can be interpreted as the most balanced or compromise solution across all models.

Hence it becomes possible to select a solution using weights that are reflect the trust towards the different departure simulation models, which a common approach in multi-criteria decision making (Ramesh & Zionts, 2013). Factors that can influence the weight determination may be data availability or model assumptions among others. Even though the forest management goal shifts away from fire suppression at all costs, information about the possible extent and locations of forest loss due to burning in the near future is of mutual interest to forest management (Finney, 2001) and is assumed to be an additional factor for decision making. Therefore, the total burned area from occurred wildfires in landscape simulations as additional decision criterion is incorporated in the analysis.

Hereto, the LandClim simulation model is set up in line with the simulation of historical conditions (Supplementary Material B), apart from specific different parameter settings. The changes relate to replacing the focus on vegetation conditions with the focus on forest fires: the current vegetation conditions are used as initial vegetation instead of random initial vegetation. The simulation time frame is 50 years from the year 2022 instead of 1000 years in the past (as used in Keane (2012) and Snell et al. (2017)). The fire ignition probability is uniform to ensure that every part of the study area has a similar potential to have been confronted with burning neighbor cells (Finney, 2005) within the short time frame. The forest vegetation is thinned to 60% of the vegetation density in the identified management areas every decade to simulate treatment. Finally, the output of burned cells is computed. The additional selection criterion is the summed-up burned area over 50 years of simulation time.

5.2 Identification of robust solutions using commonalities in solution space

The second strategy combines objective space analysis with solution space analysis by identifying similarities within optimal trade-off solutions. With the underlying forest treatment optimization model and solutions being represented as patches of treatment areas, the obvious similarity metric is the frequency of land units that constitute the optimal contiguous patches derived with the weighted sum method. From here on, this similarity metric is referred to as unit-frequency. Every forest unit gets assigned a new property that quantifies the frequency. The new property could be used without further steps to support decision making by visual inspection of the frequencies. However, in the context of a treatment allocation optimization with a threshold and a contiguity constraint (Section 4.2), the selected solution must meet the constraints, too. Therefore, a second optimization is performed with a new benefit metric: $\beta_{frequency}$. $\beta_{frequency}$ is the number of occurrences per land unit in all optimal trade-off solutions obtained with the weighted-sum-method. This consecutive optimization maximizes $\beta_{frequency}$ while satisfying the maximum treatment area and the contiguity constraint (Section 4.2).

One main advantage of this second optimization in the scenario-based analysis of optimal trade-offs is that a single solution is produced, objectively. The second desired yet unproven advantage is that a high unit-frequency may lead to a robust solution.

5.3 Design of simulation experiment

The first step is to apply the weighted sum method with the defined optimization model (Section 4.2), a maximum treatment area T of 100 hectares on the three study areas. The weights increment per step by 0.01 (minimum = 0, maximum = 1). This approach includes the single-objective optima per scenario (weight combinations (1,0,0), (0,1,0) and (0,0,1).

The resulting set of optimal solutions is used for the scenario-based objective and solution space analysis (Section 5.2). This analysis includes the burned area derived from wildfire simulations on every optimal solution with applied management activities. First, the departure indicator distributions and the optimal solutions are explored. The spatial distribution of the departure indicator variables and the optimal solutions per scenario are inspected. The exploratory analysis includes a cross-evaluation of each model's optimal configuration to the other two optimal map configurations, e.g. compute the ForSys and LANDFIRE objective value for the optimal LandClim solution. Then, robust solutions are identified using the two strategies (Section 5.2). Following the first strategy, all optimal trade-offs are shown on the trade-off hyperplane with information on how much area was burned in the wildfire simulations. Since this strategy requires a subjective weight preference setting, we chose not to select a single solution that is robust in our subjective perception. For identifying a single robust solution, the objective second strategy is used by maximizing $\beta_{frequency}$. The robustness is evaluated using the total deviation of a solution to the optimal solutions per scenario.

6. Case studies

The study areas for this project are 302, 709, and 1327 ha subsets of the Stanislaus Forest that are divided into 57, 130 and 315 treatment units (Figure 2, yellow), respectively. These subsets were also used in previous studies in the context of minimizing departure from historical conditions (Murray et al., 2022). The subsets are part of the total dataset that represents the Stanislaus National Forest (Figure 2, dark green) and extends over 47686 ha. The precipitation is highly variable, with yearly precipitation between 13 mm and 74 mm from 1951 to 2017 and with increasing minimum temperatures of 2.63°C (Estes & Gross, 2020. The overall tree density is high and significantly higher compared to the density 100 years ago (Dolanc et al., 2014; Estes & Gross, 2020), however the density of large trees has decreased due to climate- and water stress (Estes & Gross, 2020). The area is an ideal study case since the Stanislaus National Forest has been affected by multiple severe fires in the past decades.

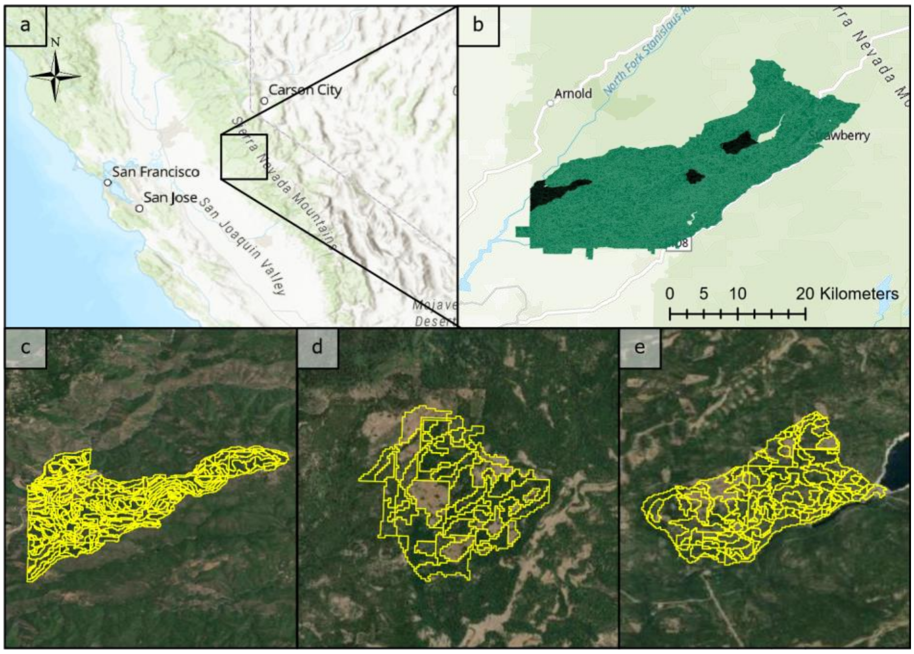

Figure 2: Stanislaus national park study area. Location in California (a), subset indications of the total National Forest (b), and study cases with 315 (c), 57 (d) and 130 (e) forest units.

7. Results

The results were obtained using a Windows computer with 16 GB RAM, i79850H Intel Processor with 6 cores and 12 logical processors. The Gurobi solver (Gurobi Optimization, LLC, 2023) was used to solve the optimization model described in Section 4.3, the remainder of the implementation is based on Python.

The three departure indicator distributions (Figure 3, upper rows per chart) show little similarity with each other. As a consequence, the optimal solutions per scenario (Figure 3, bottom rows per chart) seem highly contrary at first glance for all three study areas with mostly no overlap at all. However, as Murray and Gottsegen (1997) point out, visually dissimilar location configurations and the absence of spatial clusters should not be the only criteria to evaluate the actual similarity of solutions.

The cross-evaluation with the spatial configurations of each single objective optima with the other two objective functions (Table 1) underlines the statement: the completely different optimal spatial configurations for the LANDFIRE and LandClim scenarios yield highly similar objective values.

The optimal LANDFIRE solution is within 2.9% of the optimal solution for LandClim, and 9.9% vice versa, averaged over all study cases. The optimal solution with the ForSys scenario cross-evaluated with the LandClim only deviates by 3.3%. The optimal LANDFIRE and LandClim solutions are, however, 56.5% and 58.8% lower when evaluated with the ForSys departure indicator.

The trade-off hyperplanes (Figure 4, a, upper row) underline the observation that LANDFIRE and LandClim scenarios conflict less in comparison to the ForSys scenario: the trade-offs almost follow a linear relationship from the centre to the left on the trade-off hyperplane. That indicates a constant low trade-off between the LANDFIRE and LandClim scenarios and a constant high trade-off with the ForSys scenario.

When re-evaluating the optimal solutions with the wildfire simulation, the largest burned area is constantly approximately a third higher than the burned minimum area over the three study cases. However, there is no clear trend between the position of the trade-off hyperplane and the burned area in the wildfire simulation.

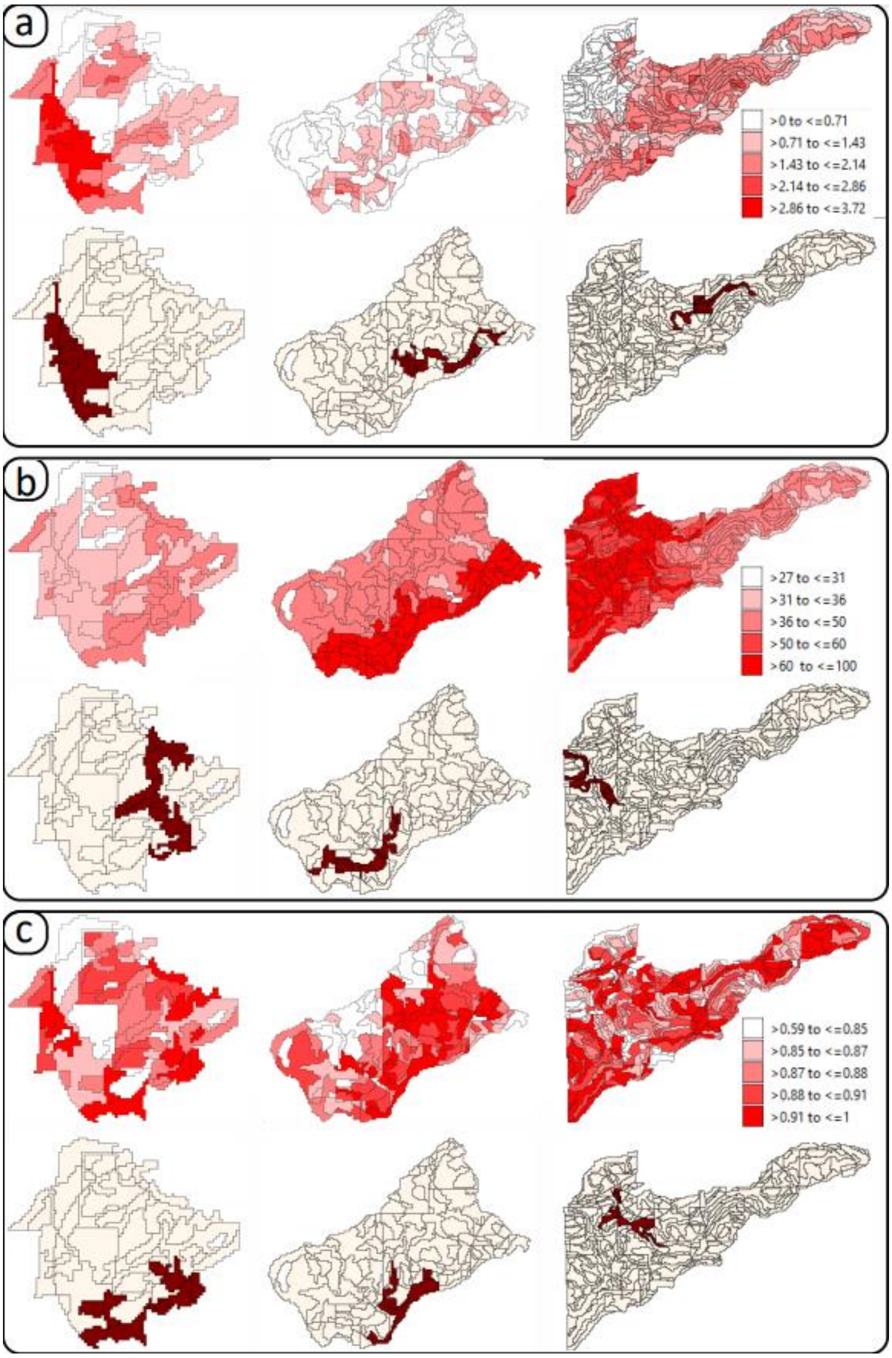

Figure 3: Departure indicator distributions (upper rows) and optima (bottom rows) for ForSys(a), LANDFIRE (b), and LandClim (c) departure indicators.

The results revealed that even though similarities between the optimal spatial configurations of the scenarios were absent, there were clear frequency patterns within optimal solutions. The frequency distributions of units that constitute optimal trade-off solutions (Figure 4, b) show consistent spatial patterns. This consistency helped to identify units that were constantly represented in optimal solutions. In all three study cases, 15-25% of the units are part of the most optimal solution per study area, with up to 70% for the 57 and 130 units study cases and up to 35% in the 315 units study case. A spatial autocorrelation analysis using Moran's I analysis (Moran, 1950) shows a significant clustering of the spatial distribution of the frequency for all three study cases.

The significant spatial clustering of frequencies indicates the presence of many commonalities in optimal solutions. Due to the presence of singular clusters in the study cases with 57 and 130 units, it is not surprising that the resulting optimal solutions that maximize $\theta_{frequency}$ (Section 5.2) (Figure 4, c) include all units with the highest unit-frequency. With two spatial clusters being present in the study case with 315 units in combination with the contiguity constraint, some of the most frequently selected units in the north are not included.

Table 1: Deviation from optima per scenario.

ForSys	57 units		130 units		315 units		
Optima	312.9		131.2		206.5		
Difference	Abs.	Rel.	Abs.	Rel.	Abs.	Rel.	Mean
LANDFIRE	-239.1	-76.4%	-21.6	-16.5%	-158.5	-76.7%	-56.5%
LandClim	-180.5	-57.7%	-87.8	-60.0%	-121.2	-58.7%	-58.8%
Robust	0	0%	-7.3	-5.5%	-55.34	-26.8%	-10.8%
LANDFIRE	57 units		130 units		315 units		
Optima	3948		7221		7681		
Difference	Abs.	Rel.	Abs.	Rel.	Abs.	Rel.	Mean
ForSys	-698.5	-17.7%	-897.5	-12.4%	-3454	-45.0%	-25.0%
LandClim	-171.1	-4.3%	-407.9	-5.7%	-1524	-19.8%	-9.9%
Robust	-698.5	-17.7%	-302.3	-4.2%	-837.7	-10.9%	-10.9%
LandClim	57 units		130 units		315 units		
Optima	89.8		91.0		91.1		
Difference	Abs.	Rel.	Abs.	Rel.	Abs.	Rel.	Mean
ForSys	-2.42	-2.7%	-3.5	-3.9%	-3.1	-3.4%	-3.32%
LANDFIRE	-0.78	-0.9%	-2.4	-2.8%	-4.5	-4.9%	-2.9%
Robust	-2.42	-2.7%	-3.4	-3.7%	-4.4	-4.7%	-3.7%

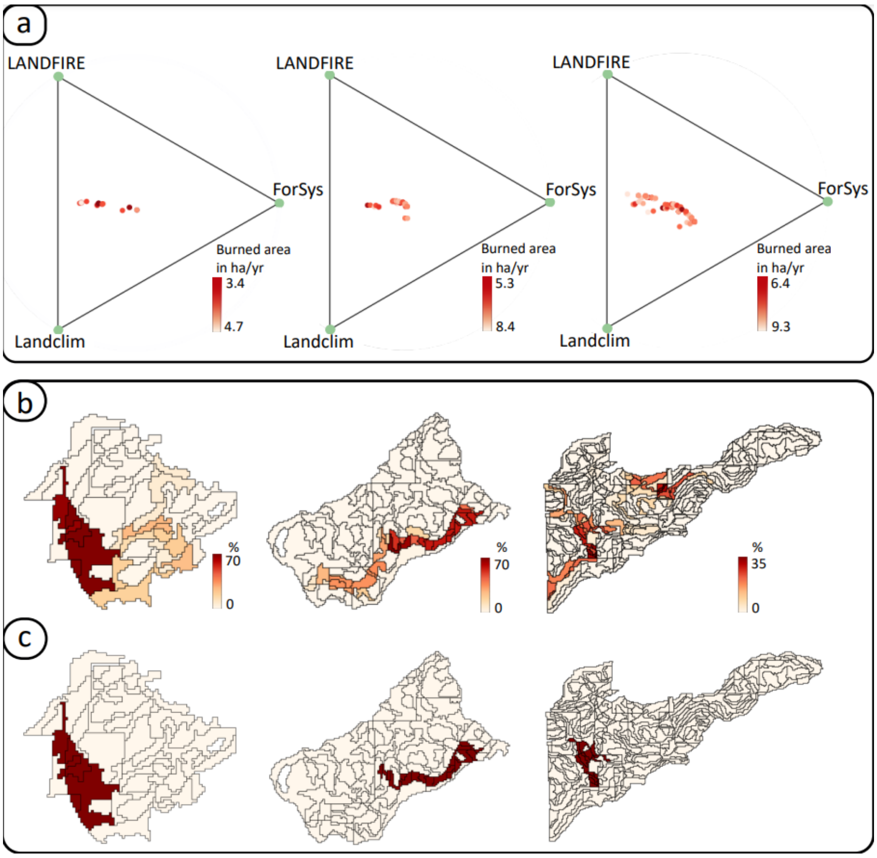

Figure 4: Trade-off hyperplane visualization of the weighted sum-method (A), percentage of optimal solutions with units being managed after the weighted sum method (B), and robust solutions from second robust strategy based on optimization maximizing the percentage shown in B (C).

Using the robustness metric of deviation towards the scenario optima's objective value, the unit-frequency maximizing strategy (second robust strategy described in Section 5.2) delivered objective robust solutions in all study cases. The objective values of the robust spatial configurations are within a percentage deviation of 11% compared to the optimal solutions per scenario (Table 1). Averaged over all scenarios, the deviation is 8.5%, which is much less than the other model deviations: The average deviation of the LandClim optima to the other scenario optima is 34.3%, the average deviation of the LANDFIRE optima is 29.2%, and the average deviation of the ForSys optima is 14.2%. On average over all scenarios, the deviation between all single scenarios is 25.9%, 17.4% higher compared to the robust solution.

The highest difference is present when evaluating the LANDFIRE and the LandClim optima under the ForSys scenario. Here, the deviation of the robust solution to the ForSys optima is approximately 40% lower.

8. Discussion

The exploratory analysis of the optimal trade-off solutions revealed that the LandClim and LANDFIRE scenarios show little conflict, whereas both scenarios conflict with the ForSys scenario. The high deviations of optimal solutions when re-evaluated with the other scenarios reveal the conflict between the optima per scenario, and the trade-off hyperplanes reveal the trade-offs between the compromise solutions, too. The conflict affirms the need for robust strategies.

The trade-off hyperplane allows a selection of solutions based on trust towards the different scenarios by defining sets of weights. The linear trade-offs on the hyperplanes, in combination with the low conflict between the LANDFIRE and LandClim scenarios, show that it suffices to select a weight for the ForSys scenario. A high weight expresses a high confidence that ForSys scenario alone depicts the departure from historical conditions most adequately, and a solution on the right of the hyper-plane would be selected. In contrast, a lower weight expresses the attitude that all three scenarios contain valuable information about uncertain historical conditions. In this case, a solution towards the center or the left on the hyper-plane could be favorable for selection. If multiple solutions can be associated with a selected set of weights (are close to each other on the trade-off hyperplane), the expected total burned area by wildfires serves as an additional decision criterion. With this additional criterion, the expected total burned area from wildfires (that is independent from the scenarios) can be reduced by up to 30%. However, this strategy requires subjective decisions.

On the other hand, the second strategy of utilizing commonalities in the solution space from the optimal solutions is objective and successfully lead to one robust solution per study case. The robust solutions are insensitive towards the three different scenarios in all study cases: the deviations to the scenario optima are constantly low, with an average deviation of 8.5%. The deviation of 26.8% for the 315 units study case is the highest deviation. However, this highest deviation is still 32-50% lower deviation than the optimal solutions of other scenarios. Also, the deviation between all single scenarios is 17.4% higher compared to the robust solutions. This indicates insensitivity to all simulation models, or, in other words: it shows that utilizing commonalities in the solution space enables identifying robust solutions.

Following up on this strategy, one can even identify robust alternative solutions with similar objective values but variations in the spatial configurations (Figure 5), which is desirable for identifying alternative solutions (Murray, 2003). Due to the high spatial auto-correlation of the frequency distributions in combination with the contiguity constraint, the local variations in the spatial configurations are limited. Even though a high variation in optimal spatial configurations does not mean that the solutions are sensitive to uncertainty (Murray, 2003), the converse argument is not necessarily true. We propose that low variation among optimal spatial configurations indicates robustness. This is a key outcome of our study and is illustrated with the following example: suppose solution A represents the true optimum, but due to uncertainty, solution B with slightly better objective values is selected as optimal instead. If solutions A and B share a large proportion of spatial units, then much of the true optimal configuration (A) is still implemented despite the uncertainty. Therefore, high spatial similarity among solutions can reduce the likelihood that uncertainty leads to the selection of entirely divergent configurations. This reduction in risk is particularly relevant in forest treatment allocation, where treating each unit demands significant effort and resources (Austin et al., 2020).

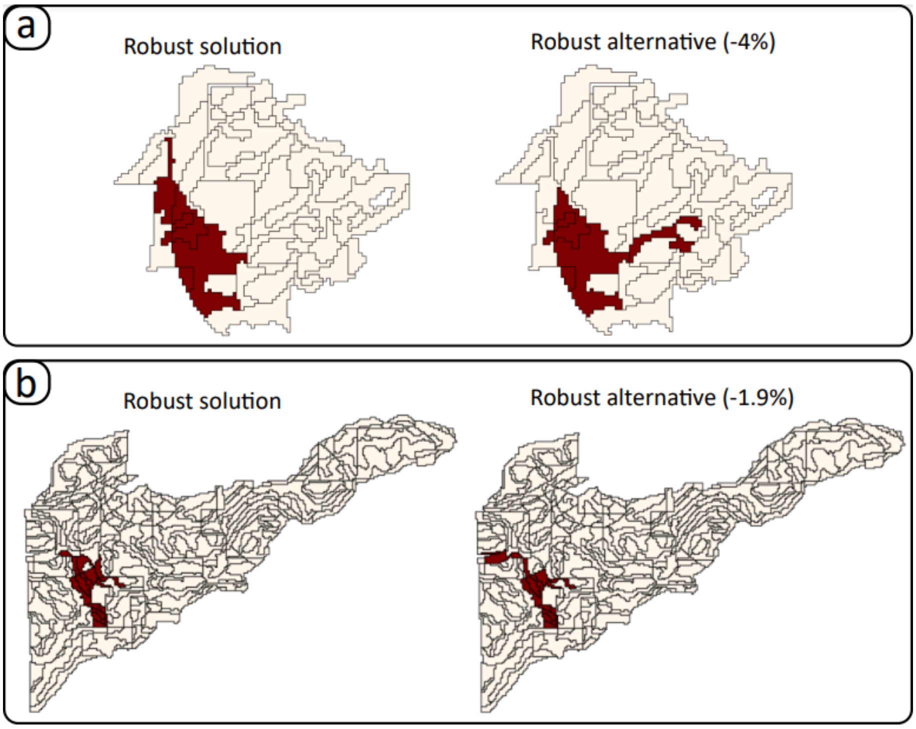

Figure 5: Robust solutions and robust alternative spatial configurations with objective value deviation.

8.1 Limitations and future work

Presented are two alternatives for identifying robust solutions under uncertainty. However, there are many options that we, as optimization modelers, had to decide upon, and each choice can alter the outcome.

First, three specific scenarios from the possible range of scenarios are used to quantify departure from historical conditions. Yet, there are many different ways to compute departure from historical conditions by using key variables other than the vegetation height and dominant vegetation class for comparing the HRV to current conditions, e.g. using another index for quantifying the HRV (Keane et al., 2011) or using different simulation models that use ecological resilience models (Cushman & McGarigal, 2019). Different sets of key variables, parameters and approaches to quantify departure lead to different scenarios, most likely resulting in different optimal solutions. Future work includes considering more scenarios, ideally as many as possible (Lempert, 2019). This extension can be done following the same method; both strategies are applicable to more scenarios.

Second, single departure indicator outputs per model are used instead of a range of outputs per model. Tree densities, tree heights, and drought levels are examples of variables that follow probability distributions in space and time. The modeling of such aleatoric uncertainties allows the quantification of uncertainty propagation. The output landscape description metrics can be assessed by repeatedly drawing samples from the probability distributions of key variables. The scenarios could then be integrated to identify robust outcomes (Maier et al., 2016). Epistemic uncertainty due to fuzzy and ambiguous data could also be addressed with the Dempster-Shafer theory (Leitmann, 1996) or using fuzzy modeling (Tanaka & Sugeno, 1998). We identify the integration of aleatoric and epistemic uncertainty modeling in scenario analysis as an important piece of future research in the context of allocation optimization under uncertainty. Here, the challenge is to specify the uncertain characteristics without having ground truth data to validate the assumptions.

Third, the changing climate affecting flora and fauna is an argument for not regaining historical conditions but formulating other desired landscape conditions (Druckenbrod et al., 2006). If future climate conditions are too different from past conditions, it is questionable that historical landscapes are the most suitable under changing conditions (Brunckhorst & Trammell, 2023). This would require a redefinition of desired landscape conditions. Following such a research direction would require dealing with the uncertain future instead of uncertain historical landscape conditions.

Fourth, the proposed unit-frequency approach is problem specific and needs adaption for other spatial optimization problems. The strategy produced robust solutions to the forest treatment allocation problem, but it cannot be applied to all location-allocation problems. One example in which the strategy cannot be applied is the identification of ideal positions under uncertainty, such as allocating fire stations to fight fire outbreaks (Bolouri et al., 2018). In such a continuous problem instance, the chance of having the same optimization output is too low to use frequencies. Different metrics describing spatial relations, such as distance-based or clustering-based metrics, are required to assess commonalities or similarities in solution space in continuous problems. Therefore, we suggest addressing additional robustness metrics that utilize different commonalities in the solution space.

8. Conclusion

In this work, we used three different departure indicators in a forest treatment optimization as scenarios to identify robust solutions under uncertainty. For the trade-off analysis between the three scenarios, the weighted sum method was used to identify the optimal weight configurations. On that basis, the trade-offs between the three simulation models and two proposed robust strategies can be analyzed. One strategy follows displaying all optimal trade-offs between the three scenarios on a trade-off hyperplane. The trade-off hyperplane, together with information about the expected total burned area enables a subjective selection of a solution that represents one's trust in the different simulation models. The second strategy is objective; it uses commonalities within optimal spatial configurations from the trade-off analysis.

The results showed that despite highly different departure indicator distributions and high trade-offs between the scenarios, robust solutions could be identified with both strategies in the study area of the Stanislaus National Park in California. The results revealed valuable insights for robust decision making by following different strategies that analyze the objective space and the solution space. The first proposed strategy results in a whole range of trade-off solutions to the three scenarios. The exploratory analysis revealed that despite visually dissimilar variable distributions, it was possible to obtain robust solutions with highly similar objective values between two of the three departure indicators.

The second robust strategy proved to identify solutions that are much more robust, over all study cases, when compared to the optimal solutions per scenario. The deviations to the scenario optima are constantly low, with an average deviation of 8.5%. Also, the deviation from all single scenarios is 17.4% higher compared to the robust solutions. The highest deviation from the robust solutions was 26.8% for the ForSys scenario in the 315 units study case. However, this highest deviation is still 32-50% lower than that of optimal solutions of other scenarios.

This study revealed one key insight for spatial explicit optimization under deep uncertainty: recognizing and utilizing common patterns in the solution space, such as the frequency with which spatial objects are included in optimal spatial configurations, facilitates the identification of robust solutions. By formalizing these commonalities in our study, through an additional optimization step that maximizes the inclusion frequency of units in optimal solutions, the risk inherent in the selection of treatment units is effectively reduced. This finding suggests that similar approaches hold promise for a variety of spatially explicit optimization problems.

Supplementary Material

The Supplementary Material can be found online at: https://sesmo.org/article/view/18725/18337.

References

Ager, A. A., Houtman, R. M., Day, M. A., Ringo, C., & Palaiologou, P. (2019). Tradeoffs between US national forest harvest targets and fuel management to reduce wildfire transmission to the wildland urban interface. Forest Ecology and Management, 434, 99–109. https://doi.org/10.1016/j.foreco.2018.12.003

Albrich, K., Rammer, W., Turner, M. G., Ratajczak, Z., Braziunas, K. H., Hansen, W. D., & Seidl, R. (2020). Simulating forest resilience: A review. Global Ecology and Biogeography, 29(12), 2082–2096. https://doi.org/10.1111/geb.13197

Austin, K. G., Baker, J. S., Sohngen, B. L., Wade, C. M., Daigneault, A., Ohrel, S. B., Ragnauth, S., & Bean, A. (2020). The economic costs of planting, preserving, and managing the world's forests to mitigate climate change. Nature

Communications, 11(1), 5946. https://doi.org/10.1038/s41467-020-19578-z

Bissonette, J. A., & Storch, I. (Eds.). (2007). Temporal dimensions of landscape ecology: Wildlife responses to variable resources (1st ed.). Springer US. https://doi.org/10.1007/978-0-387-45447-4

Bolouri, S., Vafaeinejad, A., Alesheikh, A. A., & Aghamohammadi, H. (2018). The ordered capacitated multi-objective location-allocation problem for fire stations using spatial optimization. ISPRS International Journal of Geo-Information, 7(2), Article 44. https://doi.org/10.3390/ijgi7020044

Boyer, O., Sai Hong, T., Pedram, A., Mohd Yusuff, R. B., & Zulkifli, N. (2013). A mathematical model for the industrial hazardous waste location-routing problem. Journal of Applied Mathematics, 2013, Article 435272. https://doi.org/10.1155/2013/435272

Brunckhorst, D. J., & Trammell, E. J. (2023). Future options redundancy planning: Designing multiple pathways to resilience in urban and landscape systems facing complex change. Urban Science, 7(1), Article 11. https://doi.org/10.3390/urbansci7010011

Bugmann, H. K. M. (1996). A simplified forest model to study species composition along climate gradients. Ecology, 77(7), 2055–2074. https://doi.org/10.2307/2265700

Burns, R. M., & Honkala, B. H. (Eds.). (1990). Silvics of North America: 1. Conifers; 2. Hardwoods (Vol. 2). United States Department of Agriculture, Forest Service.

Calflora. (2025). Calflora: Explore the wild plants of California. Calflora Database. Last accessed July 16, 2025, from https://www.calflora.org/

Christiansen, R. E., Lazarov, B. S., Jensen, J. S., & Sigmund, O. (2015). Creating geometrically robust designs for highly sensitive problems using topology optimization. Structural and Multidisciplinary Optimization, 52(4), 737–754. https://doi.org/10.1007/s00158-015-1265-5

Church, R. L., & Murray, A. T. (2008). Business site selection, location analysis and GIS. In Business Site Selection, Location Analysis and GIS (pp. 259–280). https://doi.org/10.1002/9780470432761

Cushman, S. A., & McGarigal, K. (2019). Metrics and models for quantifying ecological resilience at landscape scales. Frontiers in Ecology and Evolution, 7, Article 440. https://doi.org/10.3389/fevo.2019.00440

Deb, K. (2001). Multi-objective optimization using evolutionary algorithms. John Wiley & Sons.

Deb, K., & Gupta, H. (2005). Searching for robust Pareto-optimal solutions in multi-objective optimization. In C. A. Coello Coello, A. Hernández Aguirre, & E. Zitzler (Eds.), Evolutionary Multi-Criterion Optimization (pp. 150–164). Springer Berlin Heidelberg. https://doi.org/10.1007/978-3-540-31880-4_11

Dolanc, C. R., Safford, H. D., Dobrowski, S. Z., & Thorne, J. H. (2014). Twentieth century shifts in abundance and composition of vegetation types of the Sierra Nevada, CA, US. Applied Vegetation Science, 17(3), 442–455. https://doi.org/10.1111/avsc.12079

Leitmann, G. (1996). One method for robust control of uncertain systems: Theory and practice. In J. Doležal & J. Fidler (Eds.), System Modelling and Optimization (Vol. 32, Issue 1, pp. 64–74). Springer US. https://doi.org/10.1007/978-0-387-34897-1_6

Dong, S., Wang, H., Mostafavi, A., & Gao, J. (2019). Robust component: A robustness measure that incorporates access to critical facilities under disruptions. Journal of The Royal Society Interface, 16(157), 20190149. https://doi.org/10.1098/rsif.2019.0149

Druckenbrod, D. L., Dale, V. H., & Olsen, L. M. (2006). Comparing current and desired ecological conditions at a landscape scale in the Cumberland Plateau and Mountains, USA. Journal of Land Use Science, 1(2–4), 169–189. https://doi.org/10.1080/17474230601079480

Estes, B., & Gross, S. (2021). Stanislaus National Forest Climate Summary. Last accessed July 16, 2025, from https://acconsensus.org/wp-content/uploads/2021/08/StanisalusNationalForest_ClimateSummary__2021.pdf

Finney, M. A. (2001). Design of regular landscape fuel treatment patterns for modifying fire growth and behavior. Forest Science, 47(2), 219–228. https://doi.org/10.1093/forestscience/47.2.219

Finney, M. A. (2005). The challenge of quantitative risk analysis for wildland fire. Forest Ecology and Management, 211(1–2), 97–108. https://doi.org/10.1016/j.foreco.2005.02.010

Fotheringham, A. S., Densham, P. J., & Curtis, A. (1995). The zone definition problem in location-allocation modeling. Geographical Analysis, 27(1), 60–77. https://doi.org/10.1111/j.1538-4632.1995.tb00336.x

Gabrel, V., Murat, C., & Thiele, A. (2014). Recent advances in robust optimization: An overview. European Journal of Operational Research, 235(3), 471–483. https://doi.org/10.1016/j.ejor.2013.09.036

Gurobi. (2025). Gurobi Optimizer. Gurobi Optimization, LLC. Last accessed July 16, 2025, from https://www.gurobi.com/

Grüne, C. (2024). The complexity classes of Hamming distance recoverable robust problems. In J. A. Soto & A. Wiese (Eds.), LATIN 2024: Theoretical Informatics (pp. 321–335). Springer Nature Switzerland.

Hann, W. J., Shlisky, A., Havlina, D., Schon, K., Barrett, S. W., DeMeo, T. E., Pohl, K., Menakis, J. P., Hamilton, D., Jones, J., Levesque, M., & Frame, C. K. (2008). Interagency Fire Regime Condition Class (FRCC) Guidebook (Version 1.3.0). National Interagency Fuels Technology Team. Last accessed July 16, 2025, from https://npshistory.com/publications/fire/frcc-guidebook-2008.pdf

Hassani, H., Khodaygan, S., & Ghaderi, A. (2023). Bayesian reliability-based robust design optimization of mechanical systems under both aleatory and epistemic uncertainties. Engineering Optimization, 55(4), 543–563. https://doi.org/10.1080/0305215X.2021.2014828

Hengl, T., Mendes de Jesus, J., Heuvelink, G. B. M., Ruiperez Gonzalez, M., Kilibarda, M., Blagotić, A., Shangguan, W., Wright, M. N., Geng, X., Bauer-Marschallinger, B., Guevara, M. A., Vargas, R., MacMillan, R. A., Batjes, N. H., Leenaars, J. G. B., Ribeiro, E., Wheeler, I., Mantel, S., & Kempen, B. (2017). SoilGrids250m: Global gridded soil information based on machine learning. PLOS ONE, 12(2), e0169748. https://doi.org/10.1371/journal.pone.0169748

Henne, P. D., Elkin, C. M., Reineking, B., Bugmann, H., & Tinner, W. (2011). Did soil development limit spruce (Picea abies) expansion in the Central Alps during the Holocene? Testing a palaeobotanical hypothesis with a dynamic landscape model. Journal of Biogeography, 38(5), 933–949. https://doi.org/10.1111/j.1365-2699.2010.02460.x

Hildemann, M., & Verstegen, J. A. (2021). Quantifying uncertainty in Pareto fronts arising from spatial data. Environmental Modelling & Software, 141, 105069. https://doi.org/10.1016/j.envsoft.2021.105069

Jankowski, P. (1995). Integrating geographical information systems and multiple criteria decision-making methods. International Journal of Geographical Information Systems, 9(3), 251–273. https://doi.org/10.1080/02693799508902036

Keane, R. E. (2012). Creating historical range of variation (HRV) time series using landscape modeling: Overview and issues. In Historical Environmental Variation in Conservation and Natural Resource Management (pp. 113–127). Wiley. https://doi.org/10.1002/9781118329726.ch8

Keane, R. E., Holsinger, L. M., & Pratt, S. D. (2006). Simulating historical landscape dynamics using the landscape fire succession model LANDSUM version 4.0. https://doi.org/10.2737/RMRS-GTR-171

Keane, R. E., Hessburg, P. F., Landres, P. B., & Swanson, F. J. (2009). The use of historical range and variability (HRV) in landscape management. Forest Ecology and Management, 258(7), 1025–1037. https://doi.org/10.1016/j.foreco.2009.05.035

Keane, R. E., Holsinger, L., & Parsons, R. A. (2011). Evaluating indices that measure departure of current landscape composition from historical conditions. https://doi.org/10.2737/RMRS-RP-83

Labbé, M., Thisse, J.-F., & Wendell, R. E. (1991). Sensitivity analysis in minisum facility location problems. Operations Research, 39(6), 961–969. https://doi.org/10.1287/opre.39.6.961

LANDFIRE. (2016). LANDFIRE Vegetation Departure (VDep). LANDFIRE, US Department of the Interior; US Department of Agriculture. Last accessed July 16, 2025, from http://www.landfire/viewer

Lempert, R. J. (2019). Robust decision making (RDM). In Decision Making under Deep Uncertainty (pp. 23–51). Springer International Publishing. https://doi.org/10.1007/978-3-030-05252-2_2

Lempert, R., Popper, S., & Bankes, S. (2003). Shaping the next one hundred years: New methods for quantitative, long-term policy analysis. RAND Corporation. https://doi.org/10.7249/MR1626

Maier, H. R., Guillaume, J. H. A., van Delden, H., Riddell, G. A., Haasnoot, M., & Kwakkel, J. H. (2016). An uncertain future, deep uncertainty, scenarios, robustness and adaptation: How do they fit together? Environmental Modelling & Software, 81, 154–164. https://doi.org/10.1016/j.envsoft.2016.03.014

Marchau, V. A. W. J., Walker, W. E., Bloemen, P. J. T. M., & Popper, S. W. (Eds.). (2019). Decision making under deep uncertainty. Springer International Publishing. https://doi.org/10.1007/978-3-030-05252-2

Marler, R. T., & Arora, J. S. (2010). The weighted sum method for multi-objective optimization: New insights. Structural and Multidisciplinary Optimization, 41(6), 853–862. https://doi.org/10.1007/s00158-009-0460-7

McGarigal, K., & Marks, B. J. (1995). FRAGSTATS: Spatial pattern analysis program for quantifying landscape structure. https://doi.org/10.2737/PNW-GTR-351

McGaughey, R. J., & Carson, W. W. (2003). Fusing LIDAR data, photographs, and other data using 2D and 3D visualization techniques. USDA Forest Service, Pacific Northwest Research Station. Last accessed July 16, 2025, from https://www.fs.fed.us/pnw/olympia/silv/publications/opt/488_McGaugheyCarson2003.pdf

Moran, P. A. P. (1950). Notes on continuous stochastic phenomena. Biometrika, 37(1/2), 17–23. https://doi.org/10.2307/2332142

Murray, A. T. (2003). Site placement uncertainty in location analysis. Computers, Environment and Urban Systems, 27(2), 205–221. https://doi.org/10.1016/S0198-9715(02)00016-9

Murray, A. T., & Gottsegen, J. M. (1997). The influence of data aggregation on the stability of p-median location model solutions. Geographical Analysis, 29(3), 200–213. https://doi.org/10.1111/j.1538-4632.1997.tb00957.x

Murray, A. T., Church, R. L., & Pludow, B. A. (2022). Location analytics for transitioning to fire resilient landscapes. In T. X. Bui (Ed.), Proceedings of the 55th Hawaii International Conference on System Sciences (HICSS-55) (pp. 5725–5733). University of Hawaii at Manoa. https://hdl.handle.net/10125/79989

NASA. (2020). NASADEM Merged DEM Global 1 arc second V001 [Data set]. NASA Jet Propulsion Laboratory; NASA Land Processes Distributed Active Archive Center (LP DAAC). https://doi.org/10.5067/MEASURES/NASADEM/NASADEM_HGT.001

Nayak, S. (2020). Linear programming. In Fundamentals of Optimization Techniques with Algorithms (pp. 9–70). Elsevier. https://doi.org/10.1016/B978-0-12-821126-7.00002-4

Nonaka, E., & Spies, T. A. (2005). Historical range of variability in landscape structure: A simulation study in Oregon, USA. Ecological Applications, 15(5), 1727–1746. https://doi.org/10.1890/04-0902

Ramesh, R., & Zionts, S. (2013). Multiple criteria decision making. In Encyclopedia of Operations Research and Management Science (pp. 1007–1013). Springer US. https://doi.org/10.1007/978-1-4419-1153-7_653

Rubio-Sanchez, M., Raya, L., Diaz, F., & Sanchez, A. (2016). A comparative study between RadViz and Star Coordinates. IEEE Transactions on Visualization and Computer Graphics, 22(1), 619–628. https://doi.org/10.1109/TVCG.2015.2467324

Sakawa, M., & Yano, H. (1990). Trade-off rates in the hyperplane method for multi-objective optimization problems. European Journal of Operational Research, 44(1), 105–118. https://doi.org/10.1016/0377-2217(90)90319-7

Schumacher, S., Bugmann, H., & Mladenoff, D. J. (2004). Improving the formulation of tree growth and succession in a spatially explicit landscape model. Ecological Modelling, 180(1), 175–194. https://doi.org/10.1016/j.ecolmodel.2003.12.055

Shavazipour, B., Kwakkel, J. H., & Miettinen, K. (2021). Multi-scenario multi-objective robust optimization under deep uncertainty: A posteriori approach. Environmental Modelling & Software, 144, 105134. https://doi.org/10.1016/j.envsoft.2021.105134

Shirabe, T. (2005). A model of contiguity for spatial unit allocation. Geographical Analysis, 37(1), 2–16. https://doi.org/10.1111/j.1538-4632.2005.00605.x

Snell, R. S., Peringer, A., & Bugmann, H. (2017). Integrating models across temporal and spatial scales to simulate landscape patterns and dynamics in mountain pasture-woodlands. Landscape Ecology, 32(5), 1079–1096. https://doi.org/10.1007/s10980-017-0511-1

Steele, B. M., Reddy, S. K., & Keane, R. E. (2006). A methodology for assessing departure of current plant communities from historical conditions over large landscapes. Ecological Modelling, 199(1), 53–63. https://doi.org/10.1016/j.ecolmodel.2006.06.016

Tanaka, K., & Sugeno, M. (1998). Introduction to fuzzy modeling. In Fuzzy Systems (pp. 63–89). Springer US. https://doi.org/10.1007/978-1-4615-5505-6_3

Tušar, T., & Filipič, B. (2015). Visualization of Pareto front approximations in evolutionary multiobjective optimization: A critical review and the prosection method. IEEE Transactions on Evolutionary Computation, 19(2), 225–245. https://doi.org/10.1109/TEVC.2014.2313407

USAFacts. (2022). Climate in Stanislaus County, California. USAFacts. Last accessed July 16, 2025, from https://usafacts.org/issues/climate/state/california/county/stanislaus-county

Zhang, J. (2021). Modern Monte Carlo methods for efficient uncertainty quantification and propagation: A survey. WIREs Computational Statistics, 13(5), e1539. https://doi.org/10.1002/wics.1539